CSE-547

Prob#35, chapter5

Chapter 5, Problem 35

Evaluate the sum $$S_n = \sum_{k \le n} \binom{n}{k} 2^{k-n}$$

Q. Evaluate the sum $S_n = \sum_{k \le n} \binom{n}{k} 2^{k-n}$

Let $$S_n = \sum_{k \le n} \binom{n}{k} 2^{k-n} \quad \ldots\ldots(1)$$

Consider the binomial theorem,

$$(x+y)^r = \sum_{k \le r} \binom{r}{k} x^k y^{r-k} \quad \ldots\ldots.(2)$$

Compare (1) and (2).

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We will try to get the RHS of (1) to look like the RHS of (2)

$$S_n = \sum_{k \le n} \binom{n}{k} 2^{k-n}$$

or, $$S_n = \frac{1}{2^n} \sum_{k \le r} \binom{r}{k} 2^k$$

or,

$$S_n = \frac{1}{2^n} \sum_{k \le r} \binom{r}{k} 2^k 1^{r-k}$$

$$S_n = \frac{1}{2^n} \sum_{k \le r} \binom{r}{k} 2^k 1^{r-k}$$

$$(x+y)^r = \sum_{k \le r} \binom{r}{k} x^k y^{r-k}$$

$$S_n = \frac{1}{2^n}(1+2)^n = \frac{3^n}{2^n}$$

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