Let $H(n) = J(n+1) - J(n)$. Equation (1.8) tells us that $H(2n) = 2$, and $H(2n + 1) = J(2n+2) - J(2n + 1) = (2J(n+1) - 1) - (2J(n) + 1) = 2H(n) - 2$, for all $n \geq 1$. Therefore it seems possible to prove that $H(n) = 2$ for all $n$, by induction on $n$. What’s wrong here?
Detailed Solution

From the problem, we know the following:

(1) \( H(n) = J(n+1) - J(n) \)

(2) Equation (1.8) from the textbook page 10:

\[
J(1) = 1 \\
J(2n) = 2J(n) - 1, \text{ for } n \geq 1; \\
J(2n + 1) = 2J(n) + 1, \text{ for } n \geq 1.
\]

But how do we get \( H(2n) \) and \( H(2n + 1) \) ?
Let’s check whether $H(2n) = 2$, for all $n \geq 1$ as the problem stated:

$H(2n) = J(2n + 1) - J(2n) \quad H(n) = J(n+1) - J(n)$

$= [2J(n) + 1] - [2J(n) - 1] \quad \text{Eq. (1.8)}$

$= 2J(n) + 1 - 2J(n) + 1 \quad \text{Take the } [ ] \text{ out}$

$= 2 \quad \text{YES !} \quad \text{Algebra}$

Eq. (1.8): $J(1) = 1$

$J(2n) = 2J(n) - 1, \quad \text{for } n \geq 1;$

$J(2n + 1) = 2J(n) + 1, \quad \text{for } n \geq 1.$
Detailed Solution - continue

Let’s check whether \( H(2n + 1) = J(2n+2) - J(2n + 1) = (2J(n+1) - 1) - (2J(n) + 1) = 2H(n) - 2 \), for all \( n \geq 1 \) as the problem stated:

\[
H(2n + 1) = J(2n + 1 + 1) - J(2n + 1) = J(2n + 2) - J(2n + 1) = J(2(n + 1)) - J(2n + 1) = [2J(n + 1) - 1] - [2J(n) + 1] = 2J(n + 1) - 1 - 2J(n) - 1 = 2[J(n + 1) - J(n)] - 2 = 2H(n) - 2 \quad \text{YES!}
\]

\( H(n) = J(n+1) - J(n) \)

\( H(n) = J(n+1) - J(n) \)

\( \text{Algebra} \)

\( \text{Algebra} \)

\( \text{Eq. (1.8)} \)

\( \text{Eq. (1.8)} \)

\( \text{Take the [ ] out} \)

\( \text{Algebra} \)

\( H(n) = J(n+1) - J(n) \)

Eq. (1.8):

\[
J(1) = 1
\]

\[
J(2n) = 2J(n) - 1, \quad \text{for } n \geq 1;
\]

\[
J(2n + 1) = 2J(n) + 1, \quad \text{for } n \geq 1.
\]
Now, we proved the following is true.

\[ H(2n) = 2, \text{ for all } n \geq 1; \]
\[ H(2n + 1) = 2H(n) - 2, \text{ for all } n \geq 1. \]

What is missing?
The base case when \( n = 1 \).
What is \( H(1) = ? \)
What is the base case \( H(1) = ? \)

\[
H(1) = J(1 + 1) - J(1)
= J(2) - J(1)
= [2J(1) - 1] - J(1)
= 2(1) - 1 - 1
= 0
\]

\[ H(n) = J(n+1) - J(n), \text{ when } n = 1 \]

\[ J(1) = 1 \]

\[ J(2n) = 2J(n) - 1, \text{ for } n \geq 1; \]

\[ J(2n + 1) = 2J(n) + 1, \text{ for } n \geq 1. \]
Conclusion

**Review the problem:** Let $H(n) = J(n+1) - J(n)$. Equation (1.8) tells us that $H(2n) = 2$, and $H(2n + 1) = J(2n+2) - J(2n + 1) = (2J(n+1) - 1) - (2J(n) + 1) = 2H(n) - 2$, for all $n \geq 1$. Therefore it seems possible to prove that $H(n) = 2$ for all $n$, by induction on $n$. What’s wrong here?

**Conclusion:** We proved that the following are true:

- $H(2n) = 2$, for all $n \geq 1$;
- $H(2n + 1) = 2H(n) - 2$, for all $n \geq 1$.

However, the original problem does not have the base case $H(1) = 0$, which we solved.

And, $H(1) \neq 2$ for $n = 1$, which is a counter example for $H(n) = 2$ for all $n$. Therefore, $H(n) = 2$ for all $n$ is not true.