

**cse547/ams547 ONE QUESTION Quiz 2 Spring 2017
(extra 20 points)**

NAME

ID:

ams/cs

QUESTION

Part 1: 5pts

Let $P_1(x)$, $P_2(y)$ be two predicates (relations) defined in a set $X \neq \emptyset$. Prove that for any $x, y \in X$

$$[P_1(x) \cap P_2(y)] = [P_1(x)][P_2(y)]$$

Part 2: 5pts Use the Part 1 to prove

$$\sum_{i \in I, j \in J} a_i b_j = \left(\sum_{i \in I} a_i \right) \left(\sum_{j \in J} b_j \right)$$

Part 3: 5pts Write a careful proof of $\sum_{1 \leq j < k + j \leq n} a_k = \sum_{(1 \leq k \leq n-1) \cap (1 \leq j \leq n-k)} a_k$.

Part 4: 5pts Write a detailed proof (justify all steps) of the formula

$$\sum_{1 \leq j < k \leq n} \frac{1}{k-j} = nH_n - n$$