CSE548, Spring 2006

Homework 2

Due 2/28, Tuesday

For each problem, show your complete work, not just the final answer.

Part 1. Solve four of the following problems from Chapter 5: 13, 17, 18, 35, 41, 45.

Part 2. Solve one of the following problems from Chapter 5: 59, 64, 65, 66

Part 3. Solve the following problem:

(a) Let \( m \) and \( n \) be positive integers. Assume that there are \( mn \) balls of \( m \) different colors, such that there are exactly \( n \) balls of each color. In addition, we have \( k \) bags labelled with integers \( 1, 2, \ldots, k \). How many ways are there to select \( n \) balls out of these \( mn \) balls and put them in the \( k \) bags? Find a closed formula for your answer and give a complete proof for the solution. For instance, for \( m = 3, n = k = 2 \), the answer is 21. In the following, each selection is represented by a pair \((B_1, B_2)\), where \( B_i \) denotes the balls in bag \( i \). We use \( W, B, R \) to indicate the three colors, and \(-\) for an empty bag.

\[
(-, WW), \quad (-, WR), \quad (-, WB), \quad (-, RR), \quad (-, RB), \quad (-, BB), \\
(WW, -), \quad (WR, -), \quad (WB, -), \quad (RR, -), \quad (RB, -), \quad (BB, -), \\
(W, W), \quad (W, R), \quad (W, B), \quad (R, W), \quad (R, R), \quad (R, B), \quad (B, W), \quad (B, R), \quad (B, B) .
\]

Note that the two bags are distinguishable, and so \((W, R)\) and \((R, W)\) are two different selections.

(b) Can you solve the problem if the bags are indistinguishable (i.e., \((W, R)\) and \((R, W)\) are considered as the same selection, and so for \( m = 3 \) and \( n = k = 2 \), the answer is 12)? If not, how about the case with a fixed \( k = 2 \)?