Homework 1

Due 2/14, Tuesday

For each problem, show your complete work, not just the final answer.

Part 1. Solve three of the following problems from Chapter 1: 10, 15, 16, 17, 20.

Part 2. Solve four of the following problems from Chapter 2: 13, 19, 21, 23, 28, 29.

Part 3. Consider the following function on positive integers: \( f(n) = \) the number of subsets \( S \subseteq \{1, 2, \ldots, n\} \) such that the sum of all integers in \( S \) is equal to the sum of all integers in \( \{1, 2, \ldots, n\} - S \). For instance, \( f(4) = 2 \) because \( S_1 = \{1, 4\} \) and \( S_2 = \{2, 3\} \) are the two subsets of \( \{1, 2, 3, 4\} \) that have the required property.

We note that if we search over all possible subsets \( S \) of \( \{1, 2, \ldots, n\} \), it would take about \( 2^n \) steps. Show that there is a faster way to find the solutions (within \( O(n^3) \) steps). [Hint: First generalize the function into a two-parameter function, and then look for a recurrence equation for the new function. You don’t need to solve the recurrence.]