There are 4 Questions, each question is 25pts
Read carefully all of them and write solutions on next pages in spaces provided
Useful Formulas and Theorems sheet is attached

QUESTION 1 Use repertoire method to evaluate a closed formula for

\[ S_n = \sum_{k=0}^{n} (-1)^k k^2 \]

Part 1 Generalize the Problem to a 4 parameter recurrence and write the standard general form of closed form formula for it.

Part 2 1. Use functions: \( R(n) = 1 \), \( R(n) = (-1)^n \), \( R(n) = (-1)^n n \), \( R(n) = (-1)^n n^2 \), for all \( n \in \mathbb{N} \)
   to evaluate all components of the closed form formula.
   
   2. Evaluate \( S_n \) as a particular case of the general formula.

QUESTION 2 Evaluate the sum

\[ S_n = \sum_{k=0}^{n} k5^k \]

by using the following 3 methods : perturbation method, multiple sum, summation by parts Write down, in each case, which method are you using.

QUESTION 3

Part 1 Use summation by parts to evaluate \( \sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)} \)

Part 2 Evaluate

\[ \sum_{0 \leq k < n} k^m H_k \]

QUESTION 4 Prove that

\[ \lim_{n \to \infty} \frac{c^n}{n!} = 0, \text{ for } c > 0 \]
**QUESTION 1** Use repertoire method to evaluate a closed formula for

\[ S_n = \sum_{k=0}^{n} (-1)^k k^2 \]

**PART 1** Generalize the Problem to a 4 parameter recurrence and write the standard general form of closed form formula for it.

The recurrence form of the summation is: \( S_0 = \), \( S_n = \)

4 parameter recurrence is:

General form of CLOSED Formula is:

**PART 2 1.** Use functions: \( R(n) = 1 \), \( R(n) = (-1)^n \), \( R(n) = (-1)^n n \), \( R(n) = (-1)^n n^2 \), for all \( n \in \mathbb{N} \) to evaluate all components of the closed form formula.

2. Evaluate \( S_n \) as a particular case of the general formula.
Solution space
Solution space
**QUESTION 2** Evaluate the sum $S_n = \sum_{k=0}^{n} k5^k$ by using the following 3 methods: perturbation method, multiple sum, summation by parts. Write down, in each case, which method are you using.
Solution space
QUESTION 3

Part 1 Use summation by parts to evaluate $\sum_{k=0}^{n-1} \frac{H_k}{(k+1)(k+2)}$
Part 2 Evaluate

\[ \sum_{0 \leq k < n} k^n H_k \]
QUESTION 4

Prove that

$$\lim_{n \to \infty} \frac{c^n}{n!} = 0, \text{ for } c > 0$$
1 Useful Formulas

\[
x^m = x(x-1)\ldots(x-m+1), \quad \text{integer } m \geq 0
\]

\[
x^n = x(x+1)\ldots(x+m-1), \quad \text{integer } m \geq 0
\]

\[
x^{-m} = \frac{1}{(x+1)(x+2)\ldots(x+m)}, \quad \text{for } m > 0
\]

\[
x^{m+n} = x^n(x-m)^m, \quad \text{integers } m \text{ and } n
\]

\[
Df(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \Delta f(x) = f(x+1) - f(x)
\]

\[
D(x^m) = mx^{m-1}, \quad \Delta(x^n) = nx^{n-1}
\]

We define a shift operator: \(\hat{E}v(x) = v(x+1)\)

We proved \(\Delta(uv) = u\Delta v + \hat{E}v\Delta u\)

Summation by parts

\[
\sum u \delta v = uv - \sum \hat{E}v \delta u
\]

USEFUL THEOREMS

THEOREM 1
If the infinite sum

\[
\sum_{n=1}^{\infty} a_n \text{ converges, then } \lim_{n \to \infty} a_n = 0.
\]

THEOREM 2 (D’Alambert’s Criterium)
Any series with all its terms being positive real numbers, i.e.

\[
\sum_{n=1}^{\infty} a_n, \text{ for } a_n \geq 0, a_n \in R
\]

converges if the following condition holds:

\[
\lim_{n \to \infty} \frac{a_n}{a_{n+1}} < 1.
\]