Chapter 3
Propositional Languages

Part 1: Propositional Languages Intuitive Introduction
Part 2: Propositional Languages Formal Definitions
We define now a general notion of a propositional language. We show how to obtain, as specific cases, various languages for propositional classical logic and some non-classical logics. We assume the following:

All propositional languages contain an infinitely countable set of variables $VAR$, which elements are denoted by $a, b, c, ...$

with indices, if necessary.

All propositional languages share the general way their sets of formulas are formed.
Propositional Languages

We distinguish one propositional language from the other is the choice of its set of propositional connectives. We adopt a notation $L_{CON}$, where $CON$ stands for the set of connectives. We use a notation $L$ when the set of connectives is fixed.
Propositional Languages

For example, the language

$$\mathcal{L}_{\neg}$$

denotes a propositional language with only one $\neg$.

The language

$$\mathcal{L}_{\neg, \Rightarrow}$$

denotes that a language with two connectives $\neg$ and $\Rightarrow$ adopted as propositional connectives.

Remember: any formal language deals with symbols only and is also called symbolic language.
General Principles

Symbols for connectives do have intuitive meaning. Semantics provides a formal meaning of the connectives and is defined separately.

One language can have many semantics. Different logics can share the same language. For example: the language

\[ \mathcal{L}\{\neg, \cap, \cup, \Rightarrow\} \]

is used as a propositional language of classical and intuitionistic logics, some many-valued logics, and is extended to the language of modal logics.
General Principles

Several languages can share the same semantics. The classical propositional logic is the best example of such situation. Due to the functional dependency of classical logical connectives the languages:

\[ L\{\neg\Rightarrow\}, L\{\neg\land\}, L\{\neg\lor\}, L\{\neg,\land,\lor,\Rightarrow\}, \]

\[ L\{\neg,\land,\lor,\Rightarrow,\Leftrightarrow\}, L\{\uparrow\}, L\{\downarrow\} \]

all share the same semantics characteristic for classical propositional logic.
General Principles

The connectives have well established common names and readings, even if their semantic can differ. We use names negation, conjunction, disjunction and implication for $\neg$, $\cap$, $\cup$, $\Rightarrow$, respectively.

The connective $\uparrow$ is called alternative negation and $A \uparrow B$ reads: not both $A$ and $B$.

The connective $\downarrow$ is called joint negation and $A \downarrow B$ reads: neither $A$ nor $B$. 
Some Non-Classical Propositional Connectives

Other most common propositional connectives are modal connectives of possibility and necessity.

Standard modal symbols are □ for necessity and ♦ for possibility.

We will also use symbols C and I for modal connectives of possibility and necessity, respectively.

The formula CA, or ♦A reads: it is possible that A or A is possible and

□A reads: it is necessary that A or A is necessary.
Modal Propositional Connectives

Symbols $C$ and $I$ are used for their topological meaning in the semantics of standard modal logics $S_4$ and $S_5$.

In topology $C$ is a symbol for a set closure operation $CA$ means a closure of a set $A$.

$I$ is a symbol for a set interior operation $IA$ denotes an interior of the set $A$.

Modal logics extend the classical logic.

A modal logic languages are for example $L\{C,I,\neg,\cap,\cup,\Rightarrow\}$ or $L\{\Box,\Diamond,\neg,\cap,\cup,\Rightarrow\}$. 

Some Non-Classical Propositional Connectives

Knowledge logics also extend the classical logic by adding a new knowledge connective often denoted by $K$

A formula $KA$ reads: it is known that $A$ or $A$ is known.

A language of a knowledge logic is for example

$\mathcal{L}\{K, \neg, \cap, \cup, \Rightarrow\}$. 
Some Non-Classical Propositional Connectives

Autoepistemic logics extend classical logic by adding a believe connective, often denoted by $K$

A formula $BA$ reads: it is believed that $A$.

A language of an autoepistemic logic is for example

$$L\{ K, \neg, \cap, \cup, \Rightarrow \}.$$
Some Non-Classical Propositional Connectives

Temporal logics also extend classical logic by adding temporal connectives.

Some of temporal connectives are: $F$, $P$, $G$, $H$.

Their intuitive meanings are:

$FA$ reads $A$ is true at some future time,

$PA$ reads $A$ was true at some past time,

$GA$ reads $A$ will be true at all future times,

$H$ reads $A$ has always been true in the past.
Propositional Connectives

It is possible] to create connectives with more then one or two arguments.
We consider here only one or two argument connectives.