CSE451   EXERCISE 7

Problem 1

Given a proof system:

\[
S = (\mathcal{L}_{\neg, \Rightarrow}, \mathcal{E} = \mathcal{F}, AX = \{(A \Rightarrow A), (A \Rightarrow (\neg A \Rightarrow B))\}, \quad (r) \frac{(A \Rightarrow B)}{(B \Rightarrow (A \Rightarrow B))}).
\]

1. Prove that \( S \) is sound under classical semantics.
2. Prove that \( S \) is not sound under \( K \) semantics.
3. Write a formal proof in \( S \) with 2 applications of the rule \( (r) \).

Problem 2

Prove, by constructing a formal proof that

\[
\vdash_S ((\neg A \Rightarrow B) \Rightarrow (A \Rightarrow (\neg A \Rightarrow B))),
\]

where \( S \) is the proof system from Problem 1.

Problem 3

Given a proof system:

\[
S = (\mathcal{L}_{\cup, \Rightarrow}, \mathcal{E} = \mathcal{F}, AX = \{A_1, A_2\}, \quad R = \{(r)\})
\]

where

\[
A_1 = (A \Rightarrow (A \cup B)), \quad A_2 = (A \Rightarrow (B \Rightarrow A))
\]

and

\[
(r) \quad \frac{(A \Rightarrow B)}{(A \Rightarrow (A \Rightarrow B))}
\]

Prove that \( S \) is sound under classical semantics.

Problem 4

Determine whether \( S \) from the Problem 3 is sound or not sound under \( K \) semantics.

Problem 5

Write a formal proof \( A_1, A_2, A_3 \) in \( S \) from the Problem 3 with 2 applications of the rule \( (r) \) that starts with axiom \( A_1 \), i.e. such that \( A_1 = A_1 \).
**Problem 6**

Use results from Problem 4 to determine whether $\models_K A_3$.

**Problem 7**

Write a formal proof $A_1, A_2$ in $S$ from the Problem 3 with 1 application of the rule $(r)$ that starts with axiom $A_2$, i.e. such that $A_1 = A_2$.

**Problem 8**

Use results from Problem 3 to determine whether $\models A_2$.

**Problem 9**

Prove, by constructing a formal proof in $S$ from the Problem 3 that

$$\vdash_S (A \Rightarrow (A \Rightarrow (A \Rightarrow A)))$$.