CSE451 EXERCISE 7

Problem 1

Given a proof system:

$$S = (\mathcal{L}_{\{\neg, \Rightarrow\}}, \quad \mathcal{E} = \mathcal{F} \quad AX = \{(A \Rightarrow A), (A \Rightarrow (\neg A \Rightarrow B))\}, \quad (r) \ \frac{(A \Rightarrow B)}{(B \Rightarrow (A \Rightarrow B))}).$$

1. Prove that S is *sound* under classical semantics.

2. Prove that S is *not sound* under **K** semantics.

3. Write a formal proof in S with 2 applications of the rule (r).

Problem 2

Prove, by constructing a formal proof that

$$\vdash_S ((\neg A \Rightarrow B) \Rightarrow (A \Rightarrow (\neg A \Rightarrow B))),$$

where S is the proof system from Problem 1.

Problem 3

Given a proof system:

$$S = (\mathcal{L}_{\{\cup, \Rightarrow\}}, \quad \mathcal{E} = \mathcal{F} \quad AX = \{A1, A2\}, \quad \mathcal{R} = \{(r)\} \ ),$$

where

$$A1 = (A \Rightarrow (A \cup B)), \quad A2 = (A \Rightarrow (B \Rightarrow A))$$

and

$$(r) \ \frac{(A \Rightarrow B)}{(A \Rightarrow (A \Rightarrow B))}$$

Prove that S is *sound* under classical semantics.

Problem 4

Determine whether S from the Problem 3 is *sound* or *not sound* under **K** semantics.

Problem 5

Write a formal proof A_1, A_2, A_3 in S from the Problem 3 with 2 applications of the rule (r) that starts with axiom A1, i.e such that $A_1 = A1$.

Problem 6

Use results from Problem 4 to determine whether $\models_{\mathbf{K}} A_3$.

Problem 7

Write a formal proof A_1, A_2 in S from the Problem 3 with 1 application of the rule (r) that starts with axiom A2, i.e such that $A_1 = A2$.

Problem 8

Use results from Problem 3 to determine whether $\models A_2$.

Problem 9

Prove, by constructing a formal proof in S from the Problem 3 that

$$\vdash_S (A \Rightarrow (A \Rightarrow (A \Rightarrow (A \Rightarrow A)))).$$