

## CSE541 EXERCISE 6

**SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!**

**QUESTION 1** Given a proof system:

$$S = (\mathcal{L}_{\{\cup, \Rightarrow\}}, \mathcal{E} = \mathcal{F} \quad AX = \{A1, A2\}, \mathcal{R} = \{(r)\}),$$

where

$$A1 = (A \Rightarrow (A \cup B)), \quad A2 = (A \Rightarrow (B \Rightarrow A))$$

and

$$(r) \frac{(A \Rightarrow B)}{(A \Rightarrow (A \Rightarrow B))}$$

1. Prove that  $S$  is *sound* under classical semantics.
2. Determine whether  $S$  is *sound* or *not sound* under **K** semantics.

**K semantics** differ from Lukasiewicz's semantics only in a case on implication only. This table is:

**K-Implication**

$\Rightarrow$	F	$\perp$	T
F	T	T	T
$\perp$	$\perp$	$\perp$	T
T	F	$\perp$	T

**QUESTION 2**

1. Write a formal proof  $A_1, A_2, A_3$  in  $S$  from the QUESTION 3 with 2 applications of the rule  $(r)$  that starts with axiom A1, i.e such that  $A_1 = A1$ .
2. Use results from QUESTION 3 to determine whether  $\models_{\mathbf{K}} A_3$ .
3. Write a formal proof  $A_1, A_2$  in  $S$  from the QUESTION 3 with 1 application of the rule  $(r)$  that starts with axiom A2, i.e such that  $A_1 = A2$ .
4. Use results from QUESTION 1 to determine whether  $\models A_2$ .

**QUESTION 3** Prove, by constructing a formal proof in  $S$  from the QUESTION 1 that

$$\vdash_S (A \Rightarrow (A \Rightarrow (A \Rightarrow (A \Rightarrow A)))).$$