CSE541 EXERCISE 5

SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!

QUESTION 1 Given a proof system:

$$S = (\mathcal{L}_{\{\neg,\Rightarrow\}},\ \ \mathcal{E} = \mathcal{F}\ \ AX = \{(A \Rightarrow A), (A \Rightarrow (\neg A \Rightarrow B))\},\ \ (r)\ \frac{(A \Rightarrow B)}{(B \Rightarrow (A \Rightarrow B))}).$$

Definition: System S is sound if and only if

(i) Axioms are tautologies and

(ii) rules of inference are sound, i.e lead from all true premisses to a true conclusion.

1. Prove that S is *sound* under classical semantics.

2. Prove that S is *not sound* under **K** semantics defined as follows.

The language is the same in case of classical logic.

Connectives $\neg, \cup, \cap$ of **K** are defined as in **Ł** logic, i.e. for any $a, b \in \{F, \bot, T\}$,

$$\neg \bot = \bot, \quad \neg F = T, \quad \neg T = F,$$

$$a \cup b = max\{a, b\},$$

$$a \cap b = min\{a, b\}.$$

Implication in Kleene's logic is defined as follows.

For any $a, b \in \{F, \bot, T\}$,

$$a \Rightarrow b = \neg a \cup b.$$

QUESTION 2 Write a formal proof in S defined in Question 1 with 2 applications of the rule (r).

QUESTION 3 Prove, by constructing a formal proof that

$$\vdash_S ((\neg A \Rightarrow B) \Rightarrow (A \Rightarrow (\neg A \Rightarrow B))).$$