

**CSE541      EXERCISE 10**

**Covers Chapters 10, 11, 12** Read and learn all examples and exercises in the chapters as well!

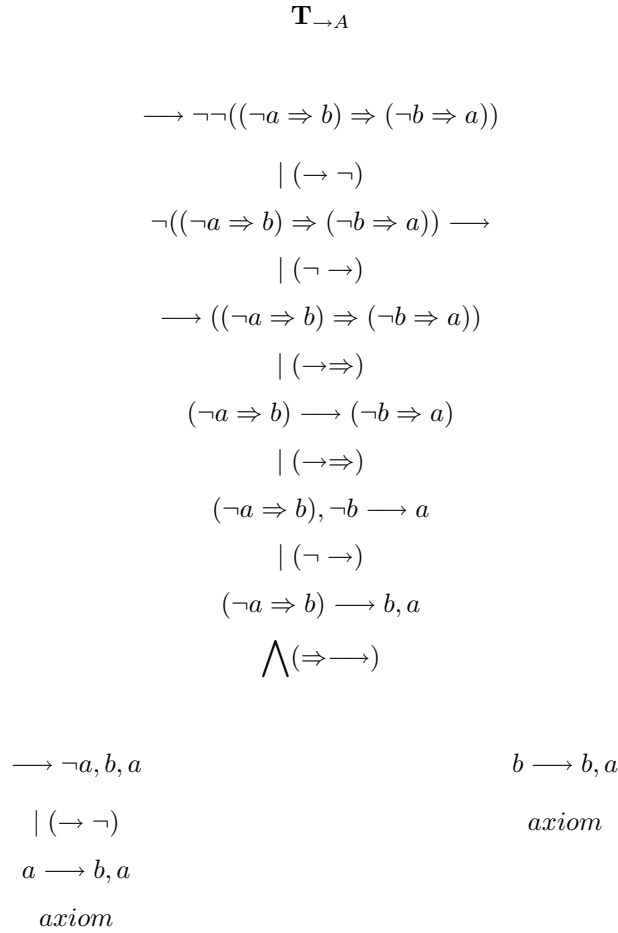
**QUESTION 1**

Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a proper decomposition tree that

- (1)  $\vdash_{\mathbf{GL}}((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$ .
- (2)  $\not\vdash_{\mathbf{GL}}((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$ .

**QUESTION 2**

Show that tree below do not constitute a proof in **GL** defined in chapter 11.



### QUESTION 3

Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a counter-model defined by a proper decomposition tree that

$$\not\models ((a \Rightarrow (\neg b \cap a)) \Rightarrow (\neg b \Rightarrow (a \cup b))).$$

### QUESTION 4

Consider a system **RS1** obtained from **RS** by changing the sequence  $\Gamma'$  into  $\Gamma$  and  $\Delta$  into  $\Delta'$  in all of the rules of inference of **RS**.

1. Construct a decomposition tree of

$$((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

2. Define in your own words, for any  $A$ , the decomposition tree  $\mathbf{T}_A$  in **RS2**.
3. Prove **Completeness Theorem** for **RS1**.

### QUESTION 5

Let **LI** be the Gentzen system for intuitionistic logic as defined in chapter 12. Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$

### QUESTION 6

We know that the formulas below are not Intuitionistic Tautologies. Verify whether **H** semantics (chapter 5) provides a counter-model for them.

$$((a \Rightarrow b) \Rightarrow (\neg a \cup b)),$$

$$((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a)).$$

### QUESTION 7

Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))$$