CSE541 EXERCISE 10

Covers Chapters 10, 11, 12 Read and learn all examples and exercises in the chapters as well!

QUESTION 1

Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a proper decomposition tree that

(1) $\vdash_{\mathbf{GL}} ((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

(2) $\nvdash_{\mathbf{GL}} ((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

QUESTION 2

Show that tree below do not constitute a proof in **GL** defined in chapter 11.

$$\mathbf{T}_{\rightarrow A}$$

$$\longrightarrow \neg\neg((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

$$\mid (\rightarrow \neg)$$

$$\neg((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)) \longrightarrow$$

$$\mid (\neg \rightarrow)$$

$$\longrightarrow ((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

$$\mid (\rightarrow \Rightarrow)$$

$$(\neg a \Rightarrow b) \longrightarrow (\neg b \Rightarrow a)$$

$$\mid (\rightarrow \Rightarrow)$$

$$(\neg a \Rightarrow b), \neg b \longrightarrow a$$

$$\mid (\neg \rightarrow)$$

$$(\neg a \Rightarrow b) \longrightarrow b, a$$

$$\bigwedge (\Rightarrow \longrightarrow)$$

$$\longrightarrow \neg a, b, a \qquad\qquad b \longrightarrow b, a$$

$$\mid (\rightarrow \neg) \qquad\qquad \textit{axiom}$$

$$a \longrightarrow b, a$$

$$\textit{axiom}$$

QUESTION 3

Let **GL** be the Gentzen style proof system for classical logic defined in chapter 11. Prove, by constructing a counter-model defined by a proper decomposition tree that

$$\not\models ((a \Rightarrow (\neg b \cap a)) \Rightarrow (\neg b \Rightarrow (a \cup b))).$$

QUESTION 4

Consider a system **RS1** obtained from **RS** by changing the sequence $\Gamma^{'}$ into Γ and Δ into $\Delta^{'}$ in all of the rules of inference of **RS**.

1. Construct a decomposition tree of

$$((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

2. Define in your own words, for any A, the decomposition tree $\mathbf{T}_A$ in **RS2**.

3. Prove **Completeness Theorem** for **RS1**.

QUESTION 5

Let **LI** be the Gentzen system for intuitionistic logic as defined in chapter 12.
Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a)).$$

QUESTION 6

We know that the formulas below are not Intuitionistic Tautologies. Verify whether **H** semantics (chapter 5) provides a counter-model for them.

$$((a \Rightarrow b) \Rightarrow (\neg a \cup b)),$$

$$((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a)).$$

QUESTION 7

Show that

$$\vdash_{\mathbf{LI}} \neg\neg((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))$$