SOLVE ALL PROBLEMS as PRACTICE and only AFTER look at the SOLUTIONS!!

Please write solutions very carefully. The grade you receive on TESTS depends not only on the fact that you SOLVE the problem, but also (30%) on elegance of your solution. Use examples from the book as a learning material of how to write solutions properly.

QUESTION 1 Describe a difference between logical and semantical paradoxes.

QUESTION 2

1. Write the following natural language statement:
   
   From the fact that it is possible that Chris is not a boy we deduce that it is not possible that Chris is not a boy or, if it is possible that Chris is not a boy, then it is not necessary that Anne is pretty.

   as a formula

   (i) \( A_1 \in \mathcal{F}_1 \) of a language \( \mathcal{L}_{\{\neg, \land, \lor, \Rightarrow\}} \),

   (ii) \( A_2 \in \mathcal{F}_2 \) of a language \( \mathcal{L}_{\{\neg, \land, \lor, \Rightarrow\}} \).

2. Degree of the formula \( A_1 \) is: , degree of the formula \( A_2 \) is:

3. List all proper, non-atomic sub-formulas of \( A_1 \).

4. List all proper, non-atomic sub-formulas of \( A_2 \) are:

5. Find a model (restricted) and a restricted counter-model for \( A_2 \) (classical semantics). Use short-hand notation. Show work.

6. There are possible counter-models restricted to \( A_2 \).

7. There possible models restricted to \( A_2 \). (Don’t need to list them, just justify your answer).

8. List 3 models and 3 counter-models for \( A_2 \) by extending the restricted model and the counter-model you have found in 4. to the set \( VAR \) of all variables.

9. There are possible models for \( A_2 \).
   There are possible counter-models for \( A_2 \).

QUESTION 3 Show that

\[ |= (\neg((a \land \neg b) \Rightarrow ((c \Rightarrow (\neg f \cup d)) \cup e)) \Rightarrow ((a \land \neg b) \land (\neg(e \Rightarrow (\neg f \cup d)) \cup e))). \]