Solve the problems and then CORRECT your solutions by comparing them with posted solutions. If you submit your corrected version - NOT a COPY of my solutions! to me Tuesday December 9 in class- you will get up do 10 extra points.

QUESTION 1
1. For the sentence
   
   \( \text{If it is not true that: } 2 + a = a + 3 \text{ and today is Monday, then: } 2 + a \neq a + 3 \text{ or today is not Monday.} \)

   write its corresponding formula \( A \). Explain your solution.

   The formula \( A \) is:

2. Define a formal language to which the formula \( A \) belongs.

   The language is:

QUESTION 2 Write the formula \( A \) from Question 1 as a formula of the language \( \mathcal{L}_{\{-,\cup}\} \), i.e. as a formula \( B \) of \( \mathcal{L}_{\{-,\cup}\} \), such that \( A \equiv B \). Write down all logical equivalences you need while solving this problem.
QUESTION 3

$H$ is the following proof system:

$$S = (\mathcal{L}_{\Rightarrow, \neg}, \ A_1, A_2, A_3, \ MP )$$

A1  $(A \Rightarrow (B \Rightarrow A))$,

A2  $((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$,

A3  $((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$,

MP  Rule of inference:

$$(MP) \frac{A; (A \Rightarrow B)}{B}$$

We know that $S$ is SOUND and COMPLETE under classical semantics.

Show whether $S$ is sound/not sound under $M$ semantics defined below.

M Negation: $\neg F = T, \neg \bot = \bot, \neg T = F$,

M Conjunction: for any $a, b \in \{F, \bot, T\}$, $a \cap b = \min\{a, b\}$,

M Disjunction: for any $a, b \in \{F, \bot, T\}$, $a \cup b = \max\{a, b\}$,

M Implication: for any $a, b \in \{F, \bot, T\}$, $a \Rightarrow b = \neg a \cup b$. 

2
**QUESTION 4**  Use the the system **RS** and its COMPLETENESS to show that

$$\models A,$$

for $A$ being the formula from the Question 1.
QUESTION 5  Consider a system RS1 obtained from RS by changing the sequence $\Gamma'$ into $\Gamma$ and $\Delta$ into $\Delta'$ in all of the rules of inference of RS.

1. Construct a decomposition tree of $A$ from the QUESTION 1 in RS1.

2. (5pts) Define in your own words, for any $A$, the decomposition tree $T_A$ in RS1.
QUESTION 6  Let $GL$ be the Gentzen style proof system for classical logic defined in chapter 10. Prove, by constructing a proper decomposition tree that

(1) $\vdash_{GL}((\neg(a \cap b) \Rightarrow b) \Rightarrow (\neg b \Rightarrow (\neg a \cup \neg b))).$

(2) PROVE that $\not\vdash_{GL}((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$. 
QUESTION 6  Show, that the formula \( \neg \neg A \), where \( A \) is the formula from Question 1 is PROVABLE in the Gentzen system LI for Intuitionistic Logic, i.e. that

\[ \vdash_{\text{LI}} \neg \neg A. \]

QUESTION 7  Let GL be the sound Gentzen style proof system for classical logic defined in chapter 10. Prove the Completeness part of the Completeness Theorem for GL.