QUESTION 1

1. For the sentence

   *If it is not true that: 2 + a = a + 3 and today is Monday, then: 2 + a ≠ a + 3 or today is not Monday.*

   write its corresponding formula $A$. Explain your solution.

   **The formula $A$ is:**

2. Define a formal language to which the formula $A$ belongs.

   **The language is:**

QUESTION 2  Write the formula $A$ from Question 1 as a formula of the language $\mathcal{L}_{\{\neg, \cup\}}$, i.e. as a formula $B$ of $\mathcal{L}_{\{\neg, \cup\}}$, such that $A \equiv B$. Write down all logical equivalences you need while solving this problem.
QUESTION 3

$H$ is the following proof system:

$$S = (\mathcal{L}_{\Rightarrow,\neg}, \ A_1, A_2, A_3, \ MP)$$

A1  $(A \Rightarrow (B \Rightarrow A))$,

A2  $((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$,

A3  $((\neg B \Rightarrow \neg A) \Rightarrow ((\neg B \Rightarrow A) \Rightarrow B))$

MP Rule of inference:

$$(MP) \quad \frac{A : (A \Rightarrow B)}{B}$$

We know that $S$ is SOUND and COMPLETE under classical semantics.

Show whether $S$ is sound/not sound under $M$ semantics defined below.

M Negation: $\neg F = T, \neg \bot = \bot, \neg T = F,$

M Conjunction: for any $a, b \in \{F, \bot, T\}, \ a \land b = \min\{a, b\},$

M Disjunction: for any $a, b \in \{F, \bot, T\}, \ a \lor b = \max\{a, b\},$

M Implication: for any $a, b \in \{F, \bot, T\}, \ a \Rightarrow b = \neg a \lor b.$
QUESTION 4 Use the system RS and its COMPLETENESS to show that

$\models A,$

for $A$ being the formula from from the Question 1.
QUESTION 5  Show, that the formula $\neg\neg A$, where $A$ is the formula from Question 1 is PROVABLE in the Gentzen system LI for Intuitionistic Logic, i.e. that

$$\vdash_{LI} \neg\neg A.$$