You will get up to 15 extra credit points for correct solution. Please submit it by Tuesday, November 1

CHALLENGE QUESTION

Consider a propositional language $L_{\{\neg, \cup, \cap, \Rightarrow\}}$ with a set $F$ of formulas.

Let $T \subseteq F$ be the set of all propositional TAUTOLOGIES under the classical semantics.

Let $S$ be a COMPLETE Hilbert proof system with for a classical propositional logic with the language $L_{\{\neg, \cup, \cap, \Rightarrow\}}$, i.e.

$T = \{A \in F : \vdash_S A\}$.

Prove that for any $A, B \in F$,

$Cn(\{A\}) \cap Cn(\{B\}) = Cn(\{(A \cup B)\}),$

where for any $X \subseteq F$ we define

$Cn(X) = \{A \in F : X \vdash_S A\}$.

HINT 1 Prove that in a Complete System $S$ Deduction Theorem holds, i.e. prove that Completeness theorem implies Deduction Theorem.

HINT 2: Use Deduction Theorem.
Space for solution