CSE371 Midterm CHALLENGE PROBLEM Fall 2011

NAME **ID:** **Math/CS**

You will get up to 15 extra credit points for correct solution. Please submit it by Tuesday, November 1

CHALLENGE QUESTION

Consider a propositional language $\mathcal{L}_{\{\neg,\cup,\cap,\Rightarrow\}}$ with a set $\mathcal{F}$ of formulas.

Let $\mathbf{T} \subseteq \mathcal{F}$ be the set of all propositional TAUTOLOGIES under the classical semantics.

Let S be a **COMPLETE Hilbert proof system** with for a classical propositional logic with the language $\mathcal{L}_{\{\neg,\cup,\cap,\Rightarrow\}}$, i.e.

$$\mathbf{T} = \{A \in \mathcal{F} : \vdash_S A\}.$$

Prove that for any $A, B \in F$,

$$Cn(\{A\}) \cap Cn(\{B\}) = Cn(\{(A \cup B)\}),$$

where for any $X \subseteq F$ we define

$$Cn(X) = \{A \in F : \ X \vdash_S A\}.$$

HINT 1 Prove that in a Complete System S Deduction Theorem holds, i.e. prove that Completeness theorem implies Deduction Theorem.

HINT 2: Use Deduction Theorem.

Space for solution