Final is due ON, or ANY TIME BEFORE Thursday, December 21, 5pm. Bring it to my office. Slide it under my door if I am not in.

**QUESTION 1** (15 pts) In all 3-valued logics semantics presented in chapter 5 we chose the language without the equivalence connective "\( \Leftrightarrow \)". Prove that in each of 3-valued logics semantics case its language can be extended to a logically equivalent language containing the equivalence connective. I.e. define \( \Leftrightarrow \) in terms of other connective and in each case provide a truth table for \( \Leftrightarrow \), and write full definition of proper semantics.

**QUESTION 2** (15 pts) For a formula \( A \) listed below construct its decomposition trees in RS and GL.

Use the proper trees to decide whether \( A \) is or is not a tautology. Write explanations. Construct a counter-model determined by the tree, if needed.

\[
A = (((a \wedge \neg c) \Rightarrow (b \cup \neg a)) \Rightarrow (\neg (b \cap c) \Rightarrow \neg a)).
\]

**QUESTION 3** (20 pts)

1. (10pts) Prove the soundness (under classical semantics) of the following rules. Explain all steps.

2. (10pts) Explain which step is not valid under the intuitionistic semantics.

   Implication rules

   \[
   \frac{(\Rightarrow)}{\Gamma, \neg A, B, \Delta} \quad \frac{(\neg \Rightarrow)}{\Gamma, A, \Delta} \quad \frac{(\neg \Rightarrow)}{\Gamma, \neg B, \Delta} \quad \frac{(\neg \Rightarrow)}{\Gamma, (A \Rightarrow B), \Delta}
   \]

**QUESTION 4** (25 pts) The algebraic models for the intuitionistic logic are defined in terms of Pseudo-Boolean Algebras in the following way.

A formula \( A \) is said to be an intuitionistic tautology if and only if \( v \models A \), for all \( v \) and all Pseudo-Boolean Algebras, where \( v \) maps \( VAR \) into universe of a Pseudo-Boolean Algebra.

I.e. \( A \) is an intuitionistic tautology if and only if it is true in all Pseudo-Boolean Algebras under all possible variable assignments.

The 3 element Heyting algebra defined in chapter 5 is a 3 element Pseudo-Boolean Algebra.
1. (10pts) Show that the 3-element Heyting algebra is a model for the following formula

\[ ((A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)). \]

2. (15pts) Use the 3-element Heyting algebra to determine whether the formulas below

\[ ((\neg A \Rightarrow B) \Rightarrow (\neg B \Rightarrow A)), \]

is not Intuitionistic Logic tautology. Explain your solution.

QUESTION 5 (20 pts) Show that

\[ \vdash_{LI} \neg (\neg (A \cap B) \Rightarrow (\neg A \cup \neg B)). \]

QUESTION 6 (20 pts) Show that

\[ \not\vdash_{LI} ((\neg A \Rightarrow \neg B) \Rightarrow (B \Rightarrow A)). \]