QUESTION 1 Give a definition and an example of a default reasoning.

QUESTION 2
1. (4pts) Write the following natural language statement:

   From the fact that it is not necessary that an elephant is not a bird we deduce that:
   it is not possible that an elephant is a bird or, if it is possible that an elephant is a bird, then it is not
   necessary that a bird flies.

   as a formula

   \[ A_1 \in \mathcal{F}_1 \text{ of a language } \mathcal{L}_{\{\neg, \wedge, \lor, \Rightarrow\}} , \]

   \[ A_2 \in \mathcal{F}_2 \text{ of a language } \mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}} . \]

2. (2pts) Main connective of the formula \( A_1 \) is: , main connective of the formula \( A_2 \) is:

3. Degree of the formula \( A_1 \) is: , degree of the formula \( A_2 \) is:
4. All proper, non-atomic sub-formulas of $A_1$ are:

5. All non-atomic sub-formulas of $A_2$ are:

   
   **A Restricted Model:**

   Evaluation:

   **A Restricted Counter-Model:**

   Evaluation:

7. There are more than 3 possible restricted counter-models of $A_2$. Justify.
8. There are more than 2 possible restricted models of $A_2$. Justify your answer.

9. List 3 models and 2 counter-models for $A_2$ by extending the restricted model and the counter-model you have found in 6. to the set $VAR$ of all variables.

10. There are possible models for $A_2$.
    There are possible counter-models for $A_2$.

QUESTION 3 Show that

\[ \models (\neg((a \land \neg b) \Rightarrow ((c \Rightarrow (\neg f \lor d)) \lor e)) \Rightarrow ((a \land \neg b) \land (\neg c \Rightarrow (\neg f \lor d)) \land \neg e))). \]
REMINDER: We define H semantics operations $\cup$ and $\cap$ as follows

$$a \cup b = \max\{a, b\}, \quad a \cap b = \min\{a, b\}.$$

The Truth Tables for Implication and Negation are:

### H-Implication

<table>
<thead>
<tr>
<th>$\Rightarrow$</th>
<th>F</th>
<th>$\bot$</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>$\bot$</td>
<td>F</td>
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<td>T</td>
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<tr>
<td>T</td>
<td>F</td>
<td>$\bot$</td>
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</tbody>
</table>

### H Negation

<table>
<thead>
<tr>
<th>$\neg$</th>
<th>F</th>
<th>$\bot$</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>$\bot$</td>
<td>T</td>
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</tbody>
</table>

QUESTION 4 We know that $v : \text{VAR} \rightarrow \{F, \bot, T\}$ is such that $v^*((a \cap b) \Rightarrow (a \Rightarrow c)) = \bot$ under H semantics. evaluate $v^*((b \Rightarrow a) \Rightarrow (a \Rightarrow \neg c)) \cup (a \Rightarrow b))$.

QUESTION 5

We define a 4 valued $L_4$ logic semantics as follows. The language is $\mathcal{L} = \mathcal{L}_{\{\neg, \Rightarrow, \cup, \cap\}}$. The logical connectives $\neg, \Rightarrow, \cup, \cap$ of $L_4$ are operations in the set $\{F, \bot_1, \bot_2, T\}$, where $\{F < \bot_1 < \bot_2 < T\}$, defined as follows.

**Negation** $\neg$ is a function $\neg : \{F, \bot_1, \bot_2, T\} \rightarrow \{F, \bot_1, \bot_2, T\}$, such that

$$\neg \bot_1 = \bot_1, \quad \neg \bot_2 = \bot_2, \quad \neg F = T, \quad \neg T = F.$$
**Conjunction** ∩ is a function ∩: \{F, \bot_1, \bot_2, T\} × \{F, \bot_1, \bot_2, T\} → \{F, \bot_1, \bot_2, T\}, such that for any \(a, b \in \{F, \bot_1, \bot_2, T\}\), \(a \cap b = \min\{a, b\}\).

**Disjunction** ∪ is a function ∪: \{F, \bot_1, \bot_2, T\} × \{F, \bot_1, \bot_2, T\} → \{F, \bot_1, \bot_2, T\}, such that for any \(a, b \in \{F, \bot_1, \bot_2, T\}\), \(a \cup b = \max\{a, b\}\).

**Implication** ⇒ is a function ⇒: \{F, \bot_1, \bot_2, T\} × \{F, \bot_1, \bot_2, T\} → \{F, \bot_1, \bot_2, T\}, such that for any \(a, b \in \{F, \bot_1, \bot_2, T\}\),

\[
a \Rightarrow b = \begin{cases} 
\neg a \cup b & \text{if } a > b \\
T & \text{otherwise}
\end{cases}
\]

**Part 1** Write all Truth Tables for \(L_4\)

**Solution:**

**Part 2** Verify whether

\[\models_{L_4}((a \Rightarrow b) \Rightarrow (\neg a \cup b))\]

**Solution:**