CSE/MAT371 EQ1 SOLUTIONS Fall 2025
(10pts)

Please **write carefully** your solutions. NO PARTIAL CREDIT. Formulas must be fully correct for credit.

QUESTION 1 (4pts)

Solution

Write the following natural language statement:

From the fact that there is a blue bird we deduce that: it is not necessary that all natural numbers are even **OR** *if it is possible that it is not true that all natural numbers are even, then it is not true that there is a blue bird.*

in the following 2 ways.

1. (2pts) As a formula $A_1 \in \mathcal{F}_1$ of a language $\mathcal{L}_{\{\neg,\, \cap,\, \cup,\, \Rightarrow\}}$. Main connective is implication. Use Propositional Variables a, b, c for the consecutive statements, where

a denotes statement: *there is a blue bird* ;

b denotes statement: *it is necessary that all natural numbers are even*,

c denotes statement: *it is possible that it is not true that all natural numbers are even*

Formula $A_1 \in \mathcal{F}_1$ is:

$$(a \Rightarrow (\neg b \cup (c \Rightarrow \neg a)))$$

2. (2pts) **Solution** As a formula $A_2 \in \mathcal{F}_2$ of a language $\mathcal{L}_{\{\neg,\, \Box,\, \Diamond,\, \cap,\, \cup,\, \Rightarrow\}}$. Use Propositional Variables a, b for consecutive statements, where

a denotes statement: *there is a blue bird*,

b denotes statement: *all natural numbers are even*

Formula $A_2 \in \mathcal{F}_2$ is:

$$(a \Rightarrow (\neg \Box b \cup (\Diamond \neg b \Rightarrow \neg a)))$$

QUESTION 2 (1pts)

Given a language $\mathcal{L} = \mathcal{L}_{\{\neg,\, \Box,\, \Diamond,\, \cap,\, \cup,\, \rightarrow\}}$ and a formula $A \in \mathcal{F}$

$$A : \quad (a \rightarrow (\neg \Box b \cup (\Diamond \neg a \rightarrow b)))$$

Solution

1. The degree of A is: 7

2. Write all sub formulas of A of the degree 0, 1.

a, b, $\Box b$, $\neg a$

3. Write all sub formulas of A of the degree 2, 3.

$\neg \Box b$, $\Diamond \neg a$, $(\Diamond \neg a \rightarrow b)$

4. Write your own, well formed formula of $\mathcal{L}$ of a degree 5.

Here are some of mine - you have yours! $\neg\neg\Box(\Box a \rightarrow b)$, $\Diamond\Diamond\Diamond\Diamond\Diamond a$, $\Box(\neg a \rightarrow (\neg\neg b \cap c))$, ...

QUSTION 3 (2pts)

Circle formulas that **are** propositional **tautologies**. Write short justification for each. CAN'T USE Truth Tables Verification Method. Use known tautologies, contradictions and Logical Equivalences.

$$\mathcal{S}_1 = \{\ ((\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e))), \qquad ((a \cap \neg b) \cup ((a \cap \neg b) \Rightarrow (\neg d \cup e))), \quad (a \cup \neg b)\ \}$$

Hint: You know that for any formulas $A, B \in \mathcal{F}$, $(A \Rightarrow B) \equiv (\neg A \cup B)$.

Solution

$\not\models (a \cup \neg b)$ as $(a \cup \neg b) = F$ for $a = F,\ b = T$

$\models ((\neg c \cap c) \Rightarrow (\neg b \Rightarrow (d \cap e)))$

because the formula $(\neg c \cap c)$ is a CONTRADICTION, i.e. always FALSE and by the definition of classical implication, $\models (A \Rightarrow B)$ for any $A, B \in \mathcal{F}$ when A is always false.

$\models ((a \cap \neg b) \cup ((a \cap \neg b) \Rightarrow (\neg d \cup e)))$

because the HINT we have that $((a \cap \neg b) \Rightarrow (\neg d \cup e)) \ \equiv\ (\neg(a \cap \neg b) \cup (\neg d \cup e))$

and we have that

$$((a \cap \neg b) \cup ((a \cap \neg b) \Rightarrow (\neg d \cup e))) \equiv ((a \cap \neg b) \cup (\neg(a \cap \neg b) \cup (\neg d \cup e))) \equiv (((a \cap \neg b) \cup \neg(a \cap \neg b)) \cup (\neg d \cup e))$$

and $\models ((a \cap \neg b) \cup \neg(a \cap \neg b))$ as a particular case of $\models (A \cup \neg A)$ for any $A \in \mathcal{F}$,

and for $A, B \in \mathcal{F}$ we have that a disjunction of a tautology A and any other formula B is a tautology.

Symbolically, $\models (A \cup B)$ when $\models A$ or $\models B$.

QUESTION 4 (4pts)

Here is a mathematical statement **S**:

For each integer $m \in Z$ the following holds: If $m > 5$, then there is a natural number $n \in N$, such that $m + n > 5$.

1. (1pts) Re-write **S** as a symbolic mathematical statement **SM** that only uses mathematical and logical symbols.

Solution **S** becomes a symbolic mathematical statement

$$\textbf{SM}: \quad \forall_{m \in Z}(m > 5 \Rightarrow \exists_{n \in N}\ m + n > 5)$$

Translate the symbolic statement **SM** into to a corresponding formula with **restricted quantifiers** using the following symbols.

Use Z(x) for $x \in Z$, N(y) for $y \in N$, a constant c for the number 5.

Write $G \in \mathbf{P}$ to denote the relation $>$, use $f \in \mathbf{F}$ to denote the function $+$.

Solution

2. (1pts) The statement $m > 5$ becomes an **atomic formula:** G(x, c).

The statement $m + n > 5$ becomes an **atomic formula** G(f(x,y), c).

4. (1pts) The symbolic mathematical statement **SM** becomes a **restricted quantifiers** formula:

$$\forall_{Z(x)}(G(x, c) \Rightarrow \exists_{N(y)} G(f(x, y), c))$$

5. (1pts) Translate your **restricted domain** quantifiers formula into a correct formula A of the predicate language $\mathcal{L}$

$$\forall x\ (\ Z(x)\ \Rightarrow (\ G(x, c) \Rightarrow \exists y\ (N(y) \cap G(f(x, y), c)\ )\ )\ )$$