

CSE/MAT371 Q6 SOLUTIONS SPRING 2024
(5pts extra credit)

ONE PROBLEM

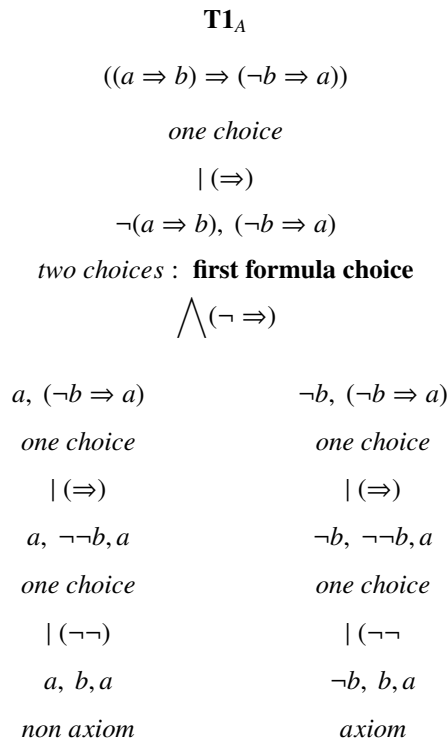
Consider a **strongly sound** system **R1** obtained from **RS** by **changing** the sequence Γ' into Γ in **all** of the rules of inference of **RS**.

1. (3pts) Construct **THREE decomposition trees** in **R1** of a formula $A: ((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

You must **INDICATE choices of decomposition** you use on each node.

2. (2pts) Use your **DECOMPOSITION TREE 1** to find a **counter model** for a **non-axiom leaf** of the tree. **Explain** why it is a **counter model** for the **formula A**.

DECOMPOSITION TREE 1 (1pts)



COUNTER MODEL and **EXPLANATION** (2pts)

The tree contains a **non- axiom** leaf, hence it is **not a proof**.

The system is **strongly sound**, so it is enough to find a counter model for a non axiom leaf as in strong sound systems **F** "climbs" the tree and the leaf counter model is also a counter model for all sequences on the branch that ends with this non axiom leaf and hence for the **formula A**.

The **counter model** for the leaf a, b, a and hence for the **formula A** is **any** $v : VAR \rightarrow \{T, F\}$, such that $v(a) = v(b) = F$.

DECOMPOSITION TREE 2 (1pts)

T2_A

$$((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

one choice

$$| (\Rightarrow)$$

$$\neg(a \Rightarrow b), (\neg b \Rightarrow a)$$

second formula choice

$$| (\Rightarrow)$$

$$\neg(a \Rightarrow b), \neg\neg b, a$$

two choices : **first formula choice**

$$\bigwedge (\neg \Rightarrow)$$

$$a, \neg\neg b, a$$

$$\neg b, \neg\neg b, a$$

$$| (\neg\neg)$$

$$| (\neg\neg)$$

$$a, b, a$$

$$\neg b, b, a$$

non axiom

axiom

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DECOMPOSITION TREE 3 (1pts)

T3_A

$((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$

| (\Rightarrow)

$\neg(a \Rightarrow b), (\neg b \Rightarrow a)$

| (\Rightarrow)

$\neg(a \Rightarrow b), \neg\neg b, a$

second formula choice

| $(\neg\neg)$

$\neg(a \Rightarrow b), b, a$

$\bigwedge (\neg \Rightarrow)$

a, b, a

$\neg b, b, a$

non axiom

axiom

COUNTER MODEL and EXPLANATION (2pts)

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