CSE/MAT371 Q6 SOLUTIONS SPRING 2024
(5pts extra credit)

ONE PROBLEM

Consider a **strongly sound** system **R1** obtained from **RS** by **changing** the sequence Γ' into Γ in **all** of the rules of inference of **RS**.

1. (3pts) Construct THREE **decomposition trees** in **R1** of a formula A: $((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$.

You must INDICATE **choices of decomposition** you use on each node.

2. (2pts) Use your DECOMPOSITION TREE 1 to find a **counter model** for a **non-axiom leaf** of the tree. **Explain** why it is a **counter model** for the **formula** A.

DECOMPOSITION TREE 1 (1pts)

$\mathbf{T1}_A$

$((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$

one choice

$\mid (\Rightarrow)$

$\neg(a \Rightarrow b),\ (\neg b \Rightarrow a)$

two choices : **first formula choice**

$\bigwedge(\neg \Rightarrow)$

$a,\ (\neg b \Rightarrow a)$	$\neg b,\ (\neg b \Rightarrow a)$
one choice	*one choice*
$\mid (\Rightarrow)$	$\mid (\Rightarrow)$
$a,\ \neg\neg b, a$	$\neg b,\ \neg\neg b, a$
one choice	*one choice*
$\mid (\neg\neg)$	$\mid (\neg\neg$
$a,\ b, a$	$\neg b,\ b, a$
non axiom	*axiom*

COUNTER MODEL and **EXPLANATION** (2pts)

The tree contains a **non- axiom** leaf, hence it is **not a proof**.

The system is **strongly sound**, so it is enough to find a counter model for a non axiom leaf as in strong sound systems **F** "climbs" the tree and the leaf counter model is also a counter model for all sequences on the branch that ends with this non axiom leaf and hence for the **formula** A.

The **counter model** for the leaf $a,\ b, a$ and hence for the **formula** A is **any** $v : VAR \longrightarrow \{T, F\}$, such that $v(a) = v(b) = F$.

DECOMPOSITION TREE 2 (1pts)

$$\mathbf{T2}_A$$

$$((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

one choice

$$\mid (\Rightarrow)$$

$$\neg(a \Rightarrow b),\ (\neg b \Rightarrow a)$$

second formula choice

$$\mid (\Rightarrow)$$

$$\neg(a \Rightarrow b),\ \neg\neg b, a$$

two choices : **first formula choice**

$$\bigwedge (\neg \Rightarrow)$$

$a,\ \neg\neg b, a$	$\neg b,\ \neg\neg b, a$
$\mid (\neg\neg)$	$\mid (\neg\neg)$
$a,\ b, a$	$\neg b,\ b, a$
non axiom	*axiom*

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DECOMPOSITION TREE 3 (1pts)

$$\mathbf{T3}_A$$

$$((a \Rightarrow b) \Rightarrow (\neg b \Rightarrow a))$$

$$\mid (\Rightarrow)$$

$$\neg(a \Rightarrow b),\ (\neg b \Rightarrow a)$$

$$\mid (\Rightarrow)$$

$$\neg(a \Rightarrow b),\ \neg\neg b, a$$

second formula choice

$$\mid (\neg\neg)$$

$$\neg(a \Rightarrow b),\ b, a$$

$$\bigwedge (\neg \Rightarrow)$$

$a,\ b, a$	$\neg b,\ b, a$
non axiom	*axiom*

COUNTER MODEL and **EXPLANATION** (2pts)

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The system is **strongly sound**, so it is enough to find a counter model for a non axiom leaf as in strong sound systems **F** "climbs" the tree and the leaf counter model is also a counter model for all sequences on the branch that ends with this non axiom leaf and hence for the **formula** A.

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