MY POINTS ARE:

TAKE test as a practice - and **correct it yourself** to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you **CAN** correct yourself- but do it **ONLY AFTER** you complete it all by yourself.

This is the goal of the PRACTICE TEST!

**PLEASE SUBMIT** your SOLUTIONS that have been **CORRECTED BY YOU** - Write corrections in RED. You **WILL GET 10 points** for THAT! even if all problems you solved were first wrong- and then CORRECTED!

Write a sum of POINTS you give yourself for your solutions - after you check your answers for corrections.

The **real midterm will have less problems**; I will make sure you will be able to complete it within 1 hour and 15 minutes.

**BRING YOUR solved-corrected TEST** to class on Monday, October 23

**PART ONE: DEFINITIONS (10pts)**

All Definitions are for language $L = L_{\{\neg,\land,\lor,\Rightarrow\}}$ and classical semantics

Write carefully the following DEFINITIONS

D1. Extentional Connectives

D2. Given the truth assignment $v : VAR \rightarrow \{T, F\}$. Write the definition of its extension $v^*$ to the set $F$ of all formulas of $L$

D3. Restricted MODEL for a given formula $A \in F$
D4. Proof System S

D5. Formal proof from \( \Gamma \) in a system S

D6. Sound rule of inference in a system S

D7. Sound proof system S

D8. Soundness and Completeness Theorem for S (classical semantics)

D9. A non-empty set \( \mathcal{G} \subseteq \mathcal{F} \) **consistent** (classical semantics)

D10. A non-empty set \( \mathcal{G} \subseteq \mathcal{F} \) **inconsistent** (classical semantics)
PART TWO: PROBLEMS (65pts)

Problem 1 (5pts), all other problems (10pts)

Problem 1

Given a mathematical statement $S$ written with logical symbols

$$(\exists x \in \mathbb{N} \ x \leq 5 \ \land \ \forall y \in \mathbb{Z} \ y = 0)$$

1. Translate $S$ into a proper logical formula that uses the restricted domain quantifiers.
2. Translate your restricted quantifiers formula into a correct formula without restricted domain quantifiers.

Write a short solution.

Problem 2

Given a formula $A : \ \forall x \exists y \ P(f(x, y), c)$ of the predicate language $L$, and two model structures $M_1 = (Z, I_1), \ M_1 = (N, I_2)$ with the interpretations defined as follows.

$P_{I_1} : = ; \ f_{I_1} : + ; \ c_{I_1} : 0$ and $P_{I_2} : > ; \ f_{I_2} : \cdot ; \ c_{I_2} : 0$.

1. Show that $M_1 \models A$
2. Show that $M_2 \not\models A$

Problem 3

$S$ is the following proof system:

$S = ( L_{\{\Rightarrow, \lor, \neg\}}, \ F, \ LA = \{(A \Rightarrow (A \lor B))\} \ (r1), \ (r2)\}$

Rules of inference:

$$(r1) \ \frac{A ; B}{(A \lor \neg B)} \quad (r2) \ \frac{A ; (A \lor B)}{B}$$
1. Verify whether $S$ is sound/not sound under classical semantics.

2. Verify whether $S$ is sound/not sound under K semantics.

3. Find a formal proof of $\neg(A \Rightarrow (A \cup B))$ in $S$, i.e. show that $\vdash_S \neg(A \Rightarrow (A \cup B))$

4. Does above point 3. prove that $\models \neg(A \Rightarrow (A \cup B))$?

Problem 4

1. Given a formula $A = ((a \cap \neg c) \Rightarrow (\neg a \cup b))$ of a language $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}}$.

Find a formula $B$ of a language $\mathcal{L}_{\{\neg, \Rightarrow\}}$, such that $A \equiv B$. List all proper logical equivalences used at each step.
2. Prove that $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}} \equiv \mathcal{L}_{\{\neg, \Rightarrow\}}$.

Problem 5

Consider the Hilbert system $H_1 := (\mathcal{L}_{\{\Rightarrow\}}, \mathcal{F}, \{A1, A2\}, (MP) \frac{A \vdash (A \Rightarrow B)}{B})$ where

$A1: (A \Rightarrow (B \Rightarrow A))$,  $A2: ((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$ and $A, B$ are any formulas from $\mathcal{F}$.

Use Deduction Theorem to prove $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_1} (A \Rightarrow C)$.

Write comments how each step was obtained.

Problem 6

Complete the steps $B_1, ..., B_5$ of the formal proof in $H_2$ of $(B \Rightarrow \neg\neg B)$ by writing all details for each step of the proof.

You can use the following already proved facts:

$F_1$ $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_2} (A \Rightarrow C)$

$F_2$ $\vdash_{H_2} (\neg\neg B \Rightarrow B)$

Here are the steps

$B_1 = ((\neg\neg B \Rightarrow \neg B) \Rightarrow ((\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B))$

$B_2 = (\neg\neg B \Rightarrow \neg B)$
\[ B_3 = ((\neg\neg B \Rightarrow B) \Rightarrow \neg\neg B) \]

\[ B_4 = (B \Rightarrow (\neg\neg B \Rightarrow B)) \]

\[ B_5 = (B \Rightarrow \neg\neg B) \]

**Problem 7** We define, for \( A, b_1, b_2, ..., b_n \) and truth assignment \( v \) a corresponding formulas \( A' \), \( B_1, B_2, ..., B_n \) as follows:

\[
A' = \begin{cases} 
A & \text{if } v^*(A) = T \\
\neg A & \text{if } v^*(A) = F
\end{cases}
\]

\[
B_i = \begin{cases} 
b_i & \text{if } v(b_i) = T \\
\neg b_i & \text{if } v(b_i) = F
\end{cases}
\]

We proved the following **Main Lemma**: For any formula \( A = A(b_1, b_2, ..., b_n) \) and any truth assignment \( v \),

if \( A', B_1, B_2, ..., B_n \) are corresponding formulas defined above, then \( B_1, B_2, ..., B_n \vdash A' \).

Let \( A \) be a formula \(((\neg a \Rightarrow \neg b) \Rightarrow (b \Rightarrow a))\), and let \( v \) be such that \( v(a) = T, \ v(b) = F \).

Write what **Main Lemma** asserts for the formula \( A \).