## cse371/mat371 LOGIC

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# CHAPTER 1 REVIEW SOME DEFINITIONS and FACTS

#### **Definition**

Logical Paradoxes, also called Logical Antinomies are paradoxes concerning the notion of a set

#### **FACT**

#### **Russell Paradox**

Consider the set A of all those sets X such that X is not a member of X

Clearly, A is a member of A if and only if A is not a member of A

So, if A is a member of A, the A is also not a member of A; and if A is not a member of A, then A is a member of A.

In any case, A is a member of A and A is not a member of A.

CONTRADICTION!



#### **FACT**

The MAIN difference between classical and intuitionists' mathematics lies in the interpretation of the word exists

In classical mathematics proving **existence** of an object x such that P(x) holds **does not mean** that one is able to indicate a method of **construction** of it

In the **intuitionists' universe** we are justified in asserting the **existence** of an object having a certain property only if we know an **effective method** for constructing, or finding such an object

#### **Definition**

Semantic Paradoxes are paradoxes that deal with the notion of truth

#### **FACT**

The Liar Paradox:

A man says: I am lying.

If he is lying, then what he says is true, and so he is not lying

If he is not lying, then what he says is not true, and so he is lying

#### CONTRADICTION!

#### **Definition**

A non-monotonic inference is a reasoning in which introduction of a new information can invalidate old facts

### Example

Consider a statement Birds fly. Tweety, we are told, is a bird.

From this, and the fact that birds fly, we conclude that Tweety can fly

This conclusion is **defeasible:** Tweety may be an ostrich, a penguin, a bird with a broken wing, or a bird whose feet have been set in concrete.

This is a non-monotonic Inference:

on learning a new fact (that Tweety has a broken wing), we are forced to **retract** our conclusion (that he could fly)



#### **Definition:**

A **default** reasoning is a reasoning that let us draw of plausible inferences from less-than- conclusive evidence in the absence of information to the contrary

Observe: non-monotonic reasoning is an example of the default reasoning

#### **Definition**

Any reasoning about one's own knowledge or **belief** is called an **auto-epistemic** reasoning

Auto-epistemic reasoning **models** the reasoning of an ideally rational agent reflecting upon his beliefs or knowledge

