CSE/MAT371 QUIZ 4 SOLUTIONS Fall 2016

QUESTION 1 (10pts)

Let S be the following proof system: $S = (\mathcal{L}_{\{\Rightarrow,\cap,\neg\}},\ \mathcal{F},\ \ LA = \{((A \cap B) \Rightarrow B)\}\ \ (r)\ \frac{A}{(B\Rightarrow A)})$,
where A, B are any formulas from $\mathcal{F}$

1. Verify whether S is **sound/not sound** under classical semantics. Use shorthand notation.

Solution: Axiom $((A \cap B) \Rightarrow B)$ is basic tautology. The rule (r) is **sound** because if we assume that A=T, we get that $(B \Rightarrow A) = T$ for any formula B. This proves that S is **sound**.

2. Prove, by constructing a **formal proof** $B_1, ...B_n$ that $\vdash_S (\neg A \Rightarrow ((A \cap A) \Rightarrow A))$.

Solution: Here is the formal proof.

B_1: $((A \cap A) \Rightarrow A)$ Axiom for B=A

B_1: $(\neg A \Rightarrow ((A \cap A) \Rightarrow A))$ rule (r) for $B = \neg A$ applied to B_1.

3. Does above point **2.** prove that $\models (\neg A \Rightarrow ((A \cap A) \Rightarrow A))$?

Solution: yes, it does because we proved in the point **2.** that the system S is sound.

QUESTION 2 (10pts)

Consider the Hilbert system $H_1 == (\mathcal{L}_{\{\Rightarrow\}},\ \mathcal{F},\ \{A1, A2\},\ (MP)\ \frac{A\ ;\ (A\Rightarrow B)}{B}\ )$ where

$A1;\ (A \Rightarrow (B \Rightarrow A)),\quad A2:\ ((A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C)))$ and A, B are any formulas from $\mathcal{F}$.

Use **Deduction Theorem** to prove $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_1} (A \Rightarrow C)$

Solution: $(A \Rightarrow B), (B \Rightarrow C) \vdash_{H_1} (A \Rightarrow C)$ if and only if (by Deduction Theorem)
$(A \Rightarrow B), (B \Rightarrow C), A \vdash_{H_1} C$

Here is a formal proof of C from $(A \Rightarrow B), (B \Rightarrow C), A$:

B_1 $(A \Rightarrow B)$ Hyp

B_2 A Hyp

B_3 B MP on B_2, B_1

B_4 $(B \Rightarrow C)$ Hyp

B_5 C MP on B_3, B_4