CSE/MAT371 QUIZ 2 Solutions Fall 2016
(20pts)

Problem 1 (6pts)

Given a formula A : $\forall x \exists y\, P(f(x,y), c)$ of the predicate language $\mathcal{L}$, and

two **model structures** $\mathbf{M_1} = (Z, I_1)$, $\mathbf{M_2} = (N, I_2)$ with the interpretations defined as follows.

$P_{I_1} : =$, $f_{I_1} : +$, $c_{I_1} : 0$ and $P_{I_2} : >$, $f_{I_2} : \cdot$, $c_{I_2} : 0$.

1. Show that $\mathbf{M_1} \models A$

A_{I_1} : $\forall_{x \in Z} \exists_{y \in Z}\, x + y = 0$ is a **true** statement;

For each $x \in Z$ exists $y = -x$ and $-x \in Z$ and $x - x = 0$.

2. Show that $\mathbf{M_2} \not\models A$

A_{I_2} : $\forall_{x \in N} \exists_{y \in N}\, x \cdot y > 0$ is a **false** statement for $x = 0$.

Problem 2 (4pts)

Consider a following set of formulas

$\mathcal{S} = \{(\forall x\, A(x) \Rightarrow \exists x\, A(x)),\quad (\forall x\, P(x,y) \Rightarrow \exists x\, P(x,y)),\quad ((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x))),$

$\exists x (A(x) \Rightarrow B) \equiv (\forall x A(x) \Rightarrow B)\}$

Circle formulas that are **predicate tautologies/ logical equivalences**.

$\not\models_p$ $((\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x\, (A(x) \cap B(x)))$,

All other formulas are are tautologies/logical equivalences.

Problem 4 (3pts)

Write a definition of the set $\mathcal{F}$ of formulas of a language $\mathcal{L}_{\{\sim, \rightarrow\}}$

Definition

The set $\mathcal{F}$ of all formulas of a propositional language $\mathcal{L}_{\{\sim, \rightarrow\}}$

is the **smallest** set for which the following conditions are satisfied.

(1) $VAR \subseteq \mathcal{F}$ (atomic formulas);

(2) If $A, B \in \mathcal{F}$, then $\sim A \in \mathcal{F}$ and $(A \rightarrow B) \in \mathcal{F}$.

Problem 3 (3pts)

Given a formula $A : (\neg \mathbf{I} \neg a \Rightarrow (\neg \mathbf{C} a \cup (\mathbf{I} a \Rightarrow \neg \mathbf{I} b)))$

1. List the main connective and the degree of the formula A.

Main connective of the formula A is: $\Rightarrow$, the degree of the formula A is: 11.

2. List all sub-formulas of A of the degree 0 and 1.

All sub-formulas of A of the degree 0 are the atomic formulas a, b.

All sub-formulas of A of the degree 1 are: $\neg a$, $\mathbf{C}a$, $\mathbf{I}a$, $\mathbf{I}b$.

Problem 3 (4pts)

. **Do not construct TTables!**

You can use the short-hand notation when justifying your answers.

1. Prove that there is only one restricted counter-model for A.

Evaluation: $(\neg a \Rightarrow (\neg b \cup (b \Rightarrow \neg c))) = F$ if and only if
$\neg a = T$ and $(\neg b \cup (b \Rightarrow \neg c)) = F$, iff
$a = F, \neg b = F$ and $(b \Rightarrow \neg c) = F$, iff
$a = F, b = T$ and $(T \Rightarrow \neg c) = F$, iff
$a = F, b = T$ and $\neg c = F$ iff
$a = F, b = T$ and $c = T$

This proves that $a = F, b = T$ and $c = T$ is the ONLY restricted counter-model for A.

2. Prove that there are 7 restricted models for A.

We have 2^3 of all possible restricted truth assignment for A; we proved that only one of them is a restricted counter-model, so there are $2^3 - 1 = 7$ restricted models for A.