Problem 1 (3pts)

1. Write the **definition** of a *semantic* paradox.

   Semantic paradoxes are paradoxes concerning the notion of truth.

2. Give an example (by name) of a *logical* paradox

   Here are 3 of them: Russel Paradox, 1902, Cantor Paradox, 1899, Burali-Forti Paradox, 1897.

3. Write the **definition** of a non-monotonic inference.

   A non-monotonic reasoning is a reasoning in which the introduction of a new information can **invalidate** old facts.

Problem 2 (5pts)

Write the following natural language statement as a formula \( A \) of our propositional language \( \mathcal{L} \).

Don’t forget to write down which propositional variables denote which basic statements.

"**From the fact** that it is **not** necessary that a red flower is not a bird we **deduce that**:

red flower is a bird or, if it is necessary that the red flower is not a bird, then the bird flies."

**Solution**

**Propositional Variables:** \( a, b, c \), where

- \( a \) denotes statement: *it is necessary that a red flower is not a bird*,
- \( b \) denotes statement: *red flower is a bird*
- \( c \) denotes statement: *the bird flies*

**Translation** The formula \( A_1 \) is: \( \neg a \Rightarrow (b \lor (a \Rightarrow c)) \)

Problem 3 (4pts)

Consider a following set of formulas

\[ S = \{(A \land B) \Rightarrow A, ((a \Rightarrow b) \land (a \Rightarrow c)) \Rightarrow (a \Rightarrow b), (A \lor \neg A), (\neg A \Rightarrow (A \Rightarrow B))\} \]

**Circle** formulas that are propositional **tautologies**. Do NOT verify.

**Solution:** all formulas is \( S \) are propositional tautologies.

Problem 4 (8pts)

Given a mathematical statement \( S \) written with logical symbols

\[ (\exists_{x \in N} x \leq 5 \land \forall_{y \in Z} y = 0) \]

1. Translate \( S \) it into a proper logical formula that **uses** the restricted domain quantifiers.
2. Translate your restricted quantifiers formula into a correct formula \textbf{without} restricted domain quantifiers.

Write a \textbf{short} solution.

\textbf{Solution}

(2pts) The corresponding \textbf{atomic formulas} of $\mathcal{L}$ are:

$N(x), \ L(x, c_1), \ Z(y), \ E(y, c_2)$, for $n \in N, \ x \leq 5, \ y \in Z, \ y = 0$, respectively.

(3pts) The statement $S$ becomes \textbf{restricted quantifiers} formula

$\exists_{N(x)} L(x, c_1) \ \cap \ \forall_{Z(y)} E(y, c_2))$

(3pts) By the \textbf{transformation rules} we get $A \in F$:

$(\exists x (N(x) \cap L(x, c_1)) \cap \forall y (Z(y) \Rightarrow E(y, c_2))).$