

**CSE/MAT371 QUIZ 1 SOLUTIONS Fall 2015**  
**(20pts)**

**PART 1: DEFINITIONS**

**D1.** Write **definition** of a LOGICAL PARADOX

**Logical Paradoxes**, also called **Logical Antinomies** are paradoxes concerning **the notion of a set**

2. Give an example (by name ) of a logical paradox

Here are 3 of them

**Russel Paradox, 1902, Cantor Paradox, 1899, Burali-Forti Paradox, 1897**

**D3.** Write **definition** of the set  $\mathcal{F}$  of **all formulas** of  $\mathcal{L}_{CON}$  for  $C_1 = \{K, L\}$  and  $C_2 = \{\cup\}$

The set of all **formulas** is defined as follows

$\mathcal{F} \subseteq \mathcal{A}^*$  and  $\mathcal{F}$  is the **smallest** set for which the following conditions are satisfied

(1)  $VAR \subseteq \mathcal{F}$  - ATOMIC FORMULAS

(2) If  $A \in \mathcal{F}$ , then  $LA, KA \in \mathcal{F}$

(3) If  $A, B \in \mathcal{F}$ , then  $(A \cap B) \in \mathcal{F}$

Write an **example** of 3 formulas of your language  $\mathcal{L}_{\{K,L,\cup\}}$

item[]  $a, La, KL(b \cup a), (KLb \cup a)$

**D3** Describe the MAIN difference between **classical** and **intuitionists' mathematics**

The **main difference** between **classical** and **intuitionists' mathematics** lies in the interpretation of the word **exists**

**PART 2: PROBLEMS**

**Problem 1** Write the following natural language statement:

*From the fact that each natural number is greater than zero we **deduce** that: it is **not possible** that Anne is a boy **or, if** it is **possible** that Anne is **not** a boy, **then** it is **necessary** that it is **not true** that each natural number is greater than zero*

in the following two ways

1. As a formula  $A_1 \in \mathcal{F}_1$  of a language  $\mathcal{L}_{\{\neg, \square, \diamond, \cap, \cup, \Rightarrow\}}$

**Propositional Variables:**  $a, b$ , where

$a$  denotes statement: *each natural number is greater than zero,*

$b$  denotes statement: *Anne is a boy*

**Propositional Modal Connectives:**  $\square, \diamond$

$\diamond$  denotes statement: **it is possible that**,  $\square$  denotes statement: **it is necessary that**

**Translation** The formula  $A_1$  is

$$(a \Rightarrow (\neg \diamond b \cup (\diamond \neg b \Rightarrow \Box \neg a)))$$

2. As a formula  $A_2 \in \mathcal{F}_2$  of a language  $\mathcal{L}_{\{\neg, \cap, \cup, \Rightarrow\}}$

**Propositional Variables:** a, b, c, d where

a denotes statement: *each natural number is greater than zero,*

b denotes statement: *possible that Anne is a boy*

c denotes statement: *possible that Anne is not a boy*

d denotes statement: *necessary that it is not true that each natural number is greater than zero*

Formula  $A_2$  is

$$(a \Rightarrow (\neg b \cup (c \Rightarrow d)))$$