QUESTION 1 (12pts)

Consider a system RS' obtained from RS by changing the sequence Γ' into Γ in all of the rules of inference of RS

Axioms are the same

1. Construct THREE different decomposition trees one for RS and TWO for RS' of a formula

\[ A = ((a \Rightarrow \neg a \cap b) \Rightarrow \neg b \Rightarrow \neg c) \]

RS tree 1 is:

\[ T_{1_A} \]
RS’ tree 2 is:

\[
\text{T}_{2A} = (a \Rightarrow (\neg a \cap b)) \Rightarrow (\neg b \Rightarrow \neg c)
\]
RS’ tree 3 is:

\[ T_{3_A} \]

\[ ((a \Rightarrow (\neg a \cap b)) \Rightarrow (\neg b \Rightarrow \neg c)) \]
2. Choose ONE of your trees to construct a counter-model v determined by that decomposition tree $T_A$

Explain WHY it is the counter-model for A

Evaluation of $v^*(A)$ is NOT proper justification!

**QUESTION 2** (8pts)

Define shortly, in your own words, for any formula $A$, its decomposition tree $T_A$ in $RS'$

Justify why your definition is correct
1 RS System

Axioms of RS
We adopt as an axiom any sequence of LITERALS which contains any propositional variable and its
negation, i.e any sequence
\[ \Gamma_1', a, \Gamma_2', \neg a, \Gamma_3', \]
\[ \Gamma_1', \neg a, \Gamma_2', a, \Gamma_3'. \]

Inference rules of RS
Disjunction rules
\[
(\cup) \quad \frac{\Gamma', A, B, \Delta}{\Gamma'', (A \cup B), \Delta}, \quad (\neg \cup) \quad \frac{\Gamma', \neg A, \Delta : \Gamma', \neg B, \Delta}{\Gamma'', \neg (A \cup B), \Delta}
\]
Conjunction rules
\[
(\cap) \quad \frac{\Gamma', A, \Delta ; \Gamma', B, \Delta}{\Gamma', (A \cap B), \Delta}, \quad (\neg \cap) \quad \frac{\Gamma', \neg A, \neg B, \Delta}{\Gamma'', \neg (A \cap B), \Delta}
\]
Implication rules
\[
(\Rightarrow) \quad \frac{\Gamma', \neg A, B, \Delta}{\Gamma'', (A \Rightarrow B), \Delta}, \quad (\neg \Rightarrow) \quad \frac{\Gamma', A, \Delta : \Gamma', \neg B, \Delta}{\Gamma'', \neg (A \Rightarrow B), \Delta}
\]
Negation rule
\[
(\neg \neg) \quad \frac{\Gamma', A, \Delta}{\Gamma'', \neg \neg A, \Delta}
\]
where \( \Gamma' \in LT^*, \Delta \in F^*, A, B \in F \).