PART 1: Definitions

Q1 (2pt)

Write a definition of a propositional language $\mathcal{L}$

Q2 (3pts)

Write the definition of the set of formulas of a language $\mathcal{L}_{\{\sim, \Rightarrow\}}$

Q3 (5pts)

Given the truth assignment (in classical semantics) $v: VAR \rightarrow \{T, F\}$

Write the definition of its extension $v^*$ to the set $\mathcal{F}$ of all formulas of $\mathcal{L}_{\{\sim, \cup\}}$
PART 2: Problems

Problem 1a (5pts)

Given a formula \( A = (\neg a \Rightarrow (\neg b \cup (b \Rightarrow \neg c))) \)

and its restricted model \( v_A : \{a, b, c\} \rightarrow \{T, F\} \), \( v_A(a) = T, v_A(b) = T, v_A(c) = F \)

Extend \( v_A \) to the set of all propositional variables \( \text{VAR} \) to obtain 2 different, non restricted models for \( A \)

Problem 1b (5pts)

1. Give an example of an infinite set of formulas of \( \mathcal{L}_{\{\neg, \cup\}} \), different from the set \( T \) of its tautologies that has a model under classical semantics

2. Give an example of an infinite set of formulas of \( \mathcal{L}_{\{\neg, \cup\}} \), different from the set \( C \) of its contradictions that is inconsistent
Problem 2  (10pts)

Definition Let $S_3$ be a 3-valued semantics for $\mathcal{L}_{\{-, \cup, \Rightarrow\}}$ defined as follows.

$$a \Rightarrow b = \neg a \cup b,$$
for any $a, b \in \{F, U, T\}$ where $F < U < T$, and

\[
\begin{array}{c|ccc}
\cup & F & U & T \\
F & F & U & T \\
U & U & U & U \\
T & T & U & T \\
\end{array}
\]

\[
\begin{array}{c|ccc}
\neg & F & U & T \\
T & F & U & T \\
\end{array}
\]

Consider the following classical tautologies: $A_1 = (a \cup \neg a)$, $A_2 = (a \Rightarrow (b \Rightarrow a))$

(a) Find $S_3$ counter-models for $A_1, A_2$, if exist.

Use shorthand notation.

(b) Define your own 2-valued semantics $S_2$ for $\mathcal{L}$, such that none of $A_1, A_2$ is a $S_2$ tautology

Verify your results. Use shorthand notation.