QUESTION 1 (10pts)

1. Write definition of restricted domain quantifiers

2. Prove de Morgan Laws for restricted domain quantifiers

List all logical equivalences you used in your proof
QUESTION 2 (5pts)

Prove that the law \((\forall x A(x) \Rightarrow \exists x A(x))\) does not hold for the restricted domain quantifiers.
QUESTION 3 (10pts)

Here is a real mathematical statement called **Pumping Lemma**

For any language \( L \),

**IF** \( L \) is infinite and regular,

**THEN** there is \( n \geq 1 \) such that for any word \( w \in L \) with lengths greater than \( n \), i.e. \(|w| \geq n\) there are \( x, y, z \in \Sigma^* \) such that \( w \) can be re-written as \( w = xyz \) and \( y \neq e \), and \( xy^n z \in L \) for all \( n \geq 0 \)

**Part 1**

Write the Pumping Lemma **symbolically** as MATHEMATICAL statement that uses logical connectives and restricted domain quantifiers symbols

Use \( \inf(L) \) to denote ” \( L \) is infinite language” and \( \text{reg}(L) \) for ”\( L \) is regular language”
Part 2

Negate carefully and fully your mathematical statement

"Fully" means that you have to negate all "inside" components applying PROPER LAWS of restricted quantifiers and propositional connectives to obtain a logically equivalent statement.
SHORT QUESTIONS (5 points)

Circle proper answer. Write one sentence justification

1. For any predicates $A(x)$, $B(x)$,
   $\neg \forall x (A(x) \cap B(x)) \equiv (\exists x \neg A(x) \cup \exists x \neg B(x))$
   JUSTIFY: 
   \[ y \ n \]

2. $\forall x (A(x) \cap B(x)) \equiv (\forall x A(x) \cap \forall x B(x))$
   JUSTIFY: 
   \[ y \ n \]

3. $\exists x (A(x) \cup B(x)) \equiv (\exists x A(x) \cup \exists x B(x))$
   JUSTIFY: 
   \[ y \ n \]

4. $\exists x (x < 1) \cup 2 + 2 = 4$ is a true statement in a set of natural numbers.
   JUSTIFY: 
   \[ y \ n \]

5. $\neg \exists n \exists x (x < \frac{1+\sigma}{n+1}) \equiv \forall n \exists x (x \geq \frac{1+\sigma}{n+1})$
   JUSTIFY: 
   \[ y \ n \]

6. $\exists x A(x) \Rightarrow \forall x A(x)$ is a predicate tautology
   JUSTIFY: 
   \[ y \ n \]

7. $\neg \forall x (A(x) \cap B(x)) \equiv (\forall x A(x) \cup \exists x \neg B(x))$
   JUSTIFY: 
   \[ y \ n \]

8. The formula $\forall x (C(x) \cap F(x))$ represents sentence: *All birds can fly* in in the domain $X \neq 0$.
   JUSTIFY: 
   \[ y \ n \]

9. For any predicates $A(x)$, $B$, (this means that $B$ does not contain the variable $x$) the formula
   $\forall x (A(x) \Rightarrow B) \Rightarrow (\exists x A(x) \Rightarrow B)$ is a predicate tautology.
   JUSTIFY: 
   \[ y \ n \]

10. $\forall x (A(x) \cup B(x)) \equiv (\forall x A(x) \cup \forall x B(x))$
    JUSTIFY: 
    \[ y \ n \]