CSE 352 – Artificial Intelligence

RESOLUTION HOMEWORK SOLUTION

1)

A) $C_1 = \{ a, \sim b \}$
$C_2 = \{ a, b, c \}$
$C_3 = \{ \sim a, c \}$
$C_4 = \{ \sim c, \sim b \}$
$CL = \{ C_1, C_2, C_3, C_4 \}$

P1: $C_1(a), C_3(\sim a)$

$$\frac{C_1(a) : C_3(\sim a)}{(C_1 - \{a\} \cup C_3 - \{\sim a\})}$$ Resolvent: $\{ \sim b, c \}$

P2: $C_2(a), C_3(\sim a)$

$$\frac{C_2(a) : C_3(\sim a)}{(C_2 - \{a\} \cup C_3 - \{\sim a\})}$$ Resolvent: $\{ b, c \}$

P3: $C_2(b), C_1(\sim b)$

$$\frac{C_2(b) : C_1(\sim b)}{(C_2 - \{b\} \cup C_1 - \{\sim b\})}$$ Resolvent: $\{ a, c \}$

P4: $C_2(b), C_4(\sim b)$

$$\frac{C_2(b) : C_4(\sim b)}{(C_2\text{-}\{b\} \cup C_4\text{-}\{\sim b\})}$$ Resolvent: { a, c, ~c

}

P5: $C_2(c), C_4(\sim c)$

$$\frac{C_2(c) : C_4(\sim c)}{(C_2\text{-}\{c\} \cup C_4\text{-}\{\sim c\})}$$ Resolvent: { a, b, ~b

}

P6: $C_3(c), C_4(\sim c)$

$$\frac{C_3(c) : C_4(\sim c)}{(C_3\text{-}\{c\} \cup C_4\text{-}\{\sim c\})}$$ Resolvent: { ~a, ~b }

B) $C_1 = \{ a, \sim a, \sim b \}$
$C_2 = \{ a, b, c \}$
$C_3 = \{ \sim a, \sim b, \sim c \}$
$C_4 = \{ b \}$
$\Delta = \{ C_1, C_2, C_3, C_4 \}$

P1: $C_1(a)$, $C_3(\sim a)$

$\underline{C_1(a) : C_3(\sim a)}$
$(C_1 - \{a\} \cup C_3 - \{\sim a\})$ Resolvent: { ~a, ~b, ~c }

P2: $C_2(a)$, $C_1(\sim a)$

$\underline{C_2(a) : C_1(\sim a)}$
$(C_2 - \{a\} \cup C_1 - \{\sim a\})$ Resolvent: { b, c, a, ~b }

P3: $C_2(a)$, $C_3(\sim a)$

$\underline{C_2(a) : C_3(\sim a)}$
$(C_2 - \{a\} \cup C_3 - \{\sim a\})$ Resolvent: { b, c, ~b, ~c }

P4: $C_2(b)$, $C_1(\sim b)$

$\underline{C_2(b) : C_1(\sim b)}$
$(C_2 - \{b\} \cup C_1 - \{\sim b\})$ Resolvent: { a, c, ~a }

P5: $C_2(b)$, $C_3(\sim b)$

$\underline{C_2(b) : C_3(\sim b)}$
$(C_2 - \{b\} \cup C_3 - \{\sim b\})$ Resolvent: $\{ a, c,$
$\sim a, \sim c \}$

P6: $C_2(c), C_3(\sim c)$

$\underline{C_2(c) : C_3(\sim c)}$
$(C_2 - \{c\} \cup C_3 - \{\sim c\})$ Resolvent: $\{ a, b,$
$\sim a, \sim b \}$

P7: $C_4(b), C_1(\sim b)$

$\underline{C_4(b) : C_1(\sim b)}$
$(C_4 - \{b\} \cup C_1 - \{\sim b\})$ Resolvent: $\{ a, \sim a \}$

P8: $C_4(b), C_3(\sim b)$

$\underline{C_4(b) : C_3(\sim b)}$
$(C_4 - \{b\} \cup C_3 - \{\sim b\})$ Resolvent: $\{ \sim a, \sim c \}$

2)

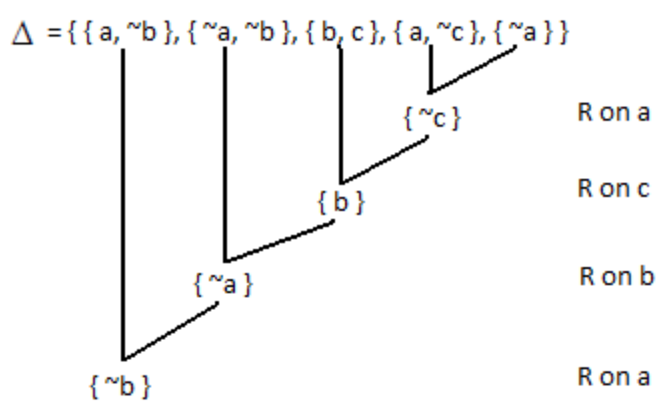
A) $C_1 = \{ a, \sim b \}$
$C_2 = \{ \sim a, \sim b \}$

$C_3 = \{ b, c \}$
$C_4 = \{ a, \sim c \}$
$C_5 = \{ \sim a \}$
$\Delta = \{ C_1, C_2, C_3, C_4, C_5 \}$

Pure literals: none
Tautologies: none

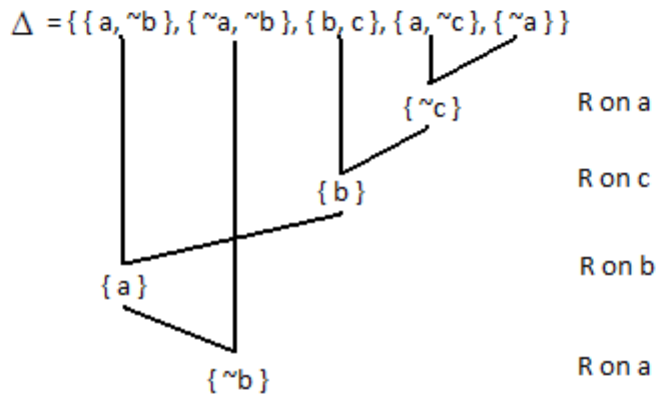
Resolution Deduction

Derivation 1:



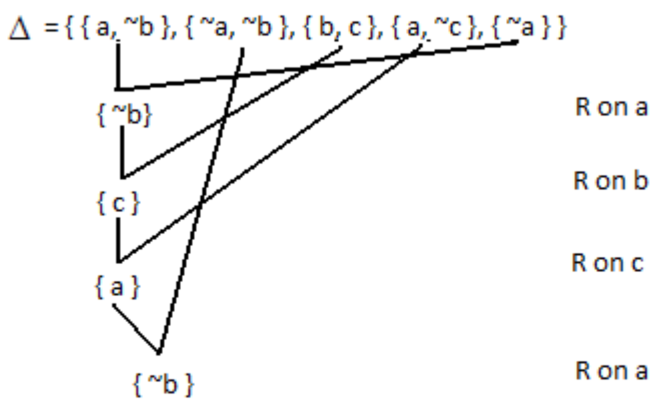
$\Delta \; \vdash_R \{ \sim b \}$

Derivation 2:



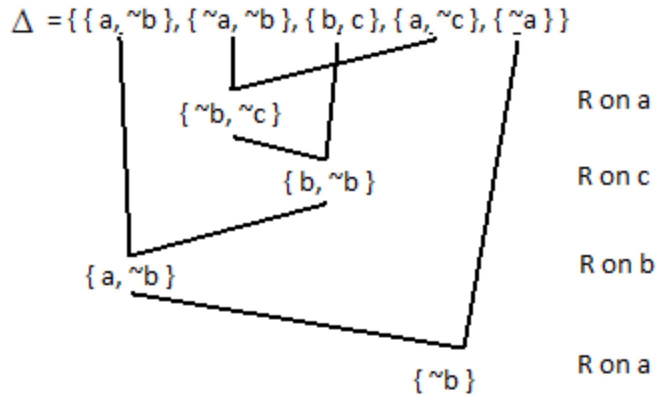
$\Delta \ |\!-_R \{\sim b\}$

Derivation 3:



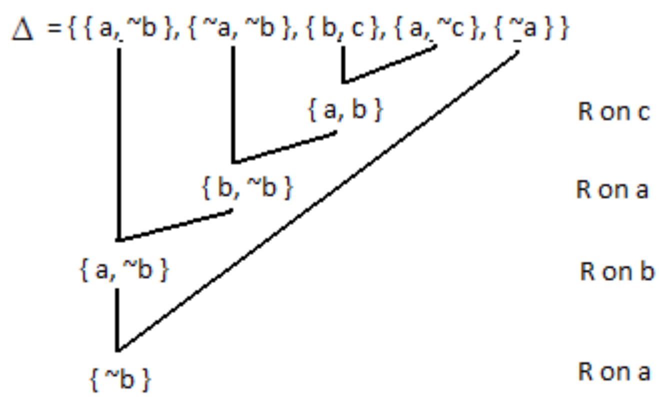
$\Delta \ |\!-_R \{\sim b\}$

Derivation 4:



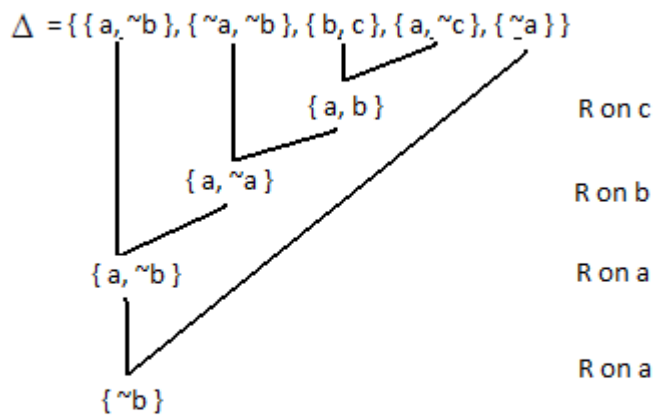
$\Delta \mid\!-_{R} \{ \sim b \}$

Derivation 5:



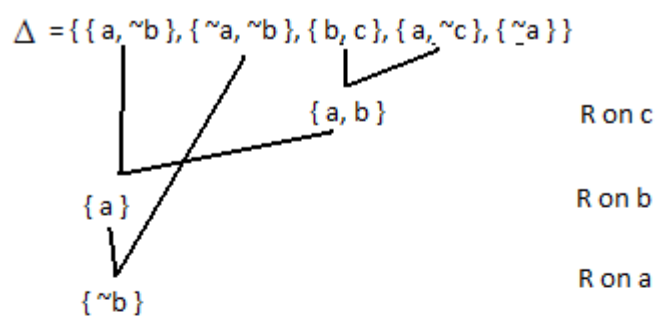
$\Delta \mid\!-_{R} \{ \sim b \}$

Derivation 6:



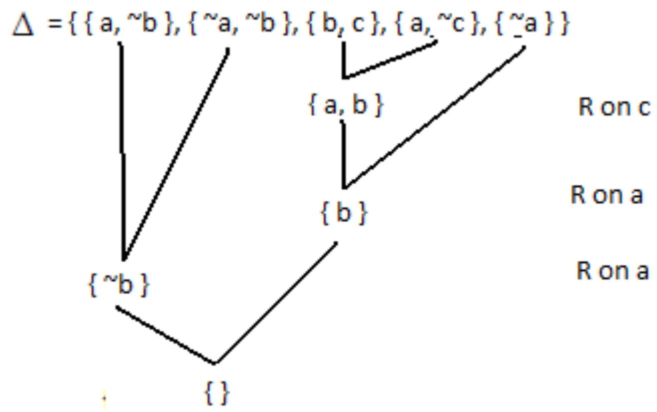
$\Delta \;|\!-_{R} \{ {\sim}b \}$

Derivation 7:



$\Delta \;|\!-_{R} \{ {\sim}b \}$

Derivation 8:



$\Delta \mid\!-_R \{\}$

Δ is UNSATISFIABLE!

B) $C_1 = \{ a, \sim b \}$
$C_2 = \{ \sim a, \sim b \}$
$C_3 = \{ b, c \}$
$C_4 = \{ a, \sim c \}$
$\Delta = \{ C_1, C_2, C_3, C_4 \}$

Pure literals: none
Tautologies: none

OBSERVE that after working on Part A of this problem, in order to resolve Δ to { }, we will need to have a pair of complimentary clauses to eliminate a literal from it. This would allow us to resolve the single literal clause with the single literal clause already given. In this part, there is no clause with a single literal. Therefore, we will need to find two pairs of complimentary clauses that resolve to complimentary and single literal clauses. The only complimentary clauses given are C_1 and C_2. Because of this, Δ is SATISFIABLE!

C) $C_1 = \{ a, b \}$
$C_2 = \{ \sim a, b \}$
$C_3 = \{ b, \sim c \}$
$C_4 = \{ a, \sim c \}$
$\Delta = \{ C_1, C_2, C_3, C_4 \}$

Pure literals: b, ~c
$\Delta' = \{ \}$

Since literals b and ~c have only one polarity in all the clauses, we can delete all clauses that contain either of them. This results in the empty set { }.
Δ is UNSATISFIABLE!

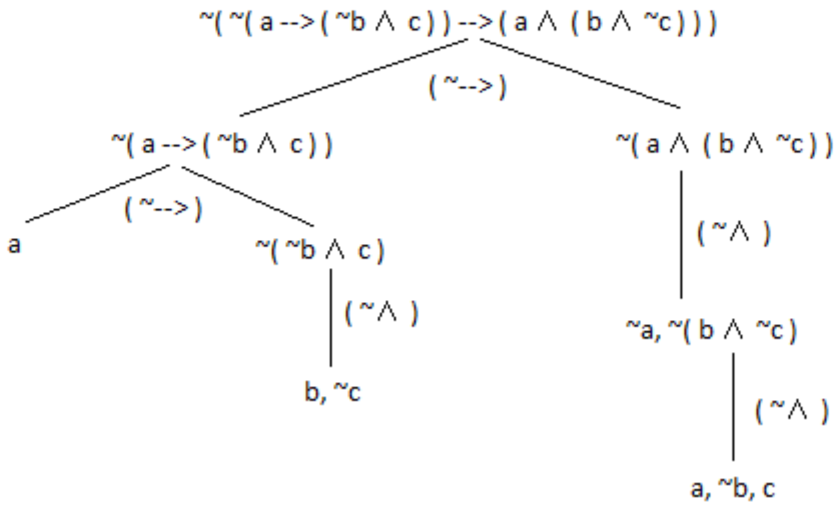
3)

A1) $(((a \rightarrow \sim b) \rightarrow (b \rightarrow \sim a)) \vee ((a \rightarrow \sim c) \vee b))$

A2) $((a \rightarrow b) \rightarrow a)$

B) $(\sim(a \rightarrow (\sim b \wedge c)) \rightarrow (a \wedge (b \wedge \sim c)))$

~B:



$\Delta_{\sim B} = \{ \{a\}, \{b, \sim c\}, \{a, \sim b, c\} \}$

A1:

(((a --> ~b) --> (b --> ~a)) V ((a --> ~c) V b))

(V)

((a --> ~b) --> (b --> ~a)) , ((a --> ~c) V b)

(-->)

~(a --> ~b), (b --> ~a), ((a --> ~c) V b)

(V)

~(a --> ~b), (b --> ~a), (a --> ~c), b

(-->)

~(a --> ~b), ~b, a, ~a, c, b

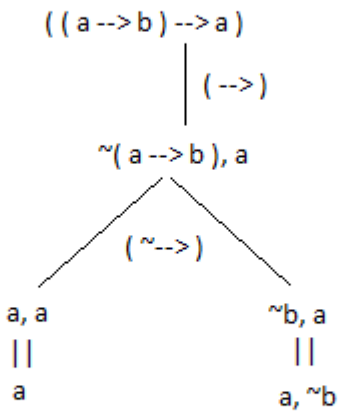
(~ -->)

a, ~b, a, ~a, c, b
||
a, ~a, b, ~b, c

b, ~b, a, ~a, c, b
||
a, ~a, b, ~b, c

Δ_{A1} = { {a, ~a, b, ~b, c} }

A2:



Δ_{A2} = { {a}, {a, ~b} }

$\Delta = \Delta_{A1} \vee \Delta_{A2} \vee \Delta_{\sim B}$ = { {a}, {b, ~c}, {a, ~b, c}, {a, ~a, b, ~b, c}, {a, ~b} }

Tautologies: {a, ~a, b, ~b, c}

Δ' = { {a}, {b, ~c}, {a, ~b, c}, {a, ~b} }

Pure literals: a

Δ'' = { {b, ~c} }

Resolution Deduction:

Cannot resolve any further.

Argument is NOT VALID!