

Introduction To Genetic Algorithms

Cse352

Artificial Intelligence

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Overview

- **Introduction** To Genetic Algorithms (GA)
- **GA Operators** and **Parameters**
- Genetic Algorithms To Solve **The Traveling Salesman Problem (TSP)**
- **Summary**

History Of Genetic Algorithms

- “**Evolutionary Computing**” was introduced in the **1960s** by **I. Rechenberg**
- **John Holland** wrote the first book on **Genetic Algorithms** ‘**Adaptation in Natural and Artificial Systems**’ in **1975**
- In **1992** **John Koza** used **genetic algorithm** to evolve programs to perform certain tasks
- He called his method “**Genetic Programming**”

What Are Genetic Algorithms?

- What exactly are Genetic Algorithms?
- As the name suggests, Genetic Algorithms borrow their basic working principle from natural genetics

Genetic Algorithms are search and optimization techniques based on **Darwin's Principle of Natural Selection**

Darwin's Principle Of Natural Selection

- IF there are organisms that **reproduce**, and
- IF offsprings **inherit traits** from their progenitors, and
- IF there is **variability of traits**, and
- IF the environment **cannot support** all members of a growing population,
- **THEN** those members of the population **with less-adaptive traits** (determined by the environment) **will die out**, and
- **THEN** those members with **more-adaptive traits** (determined by the environment) **will thrive**

The result is the **evolution of species**

Basic Idea Of Principle Of Natural Selection

**“Select The Best,
Discard The Rest”**

An Example of Natural Selection

- **Giraffes have long necks**

Giraffes with slightly longer necks **could feed on leaves of higher branches** when all lower ones had been eaten off

→ **They had a better chance of survival.**

→ **Favorable characteristic propagated through generations of giraffes.**

→ **Now, evolved species has long necks.**

NOTE: **Longer necks** may have been a deviant characteristic (**mutation**) **initially** but since it was **favorable**, was propagated over generations. Now an established trait.

So, some mutations are beneficial

Evolution Through Natural Selection

Initial Population Of Animals



Struggle For Existence-**Survival Of the Fittest**

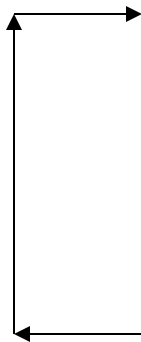


Surviving Individuals **Reproduce, Propagate** Favorable
Characteristics



Evolved Species

(Favorable Characteristic Now A Trait Of Species)



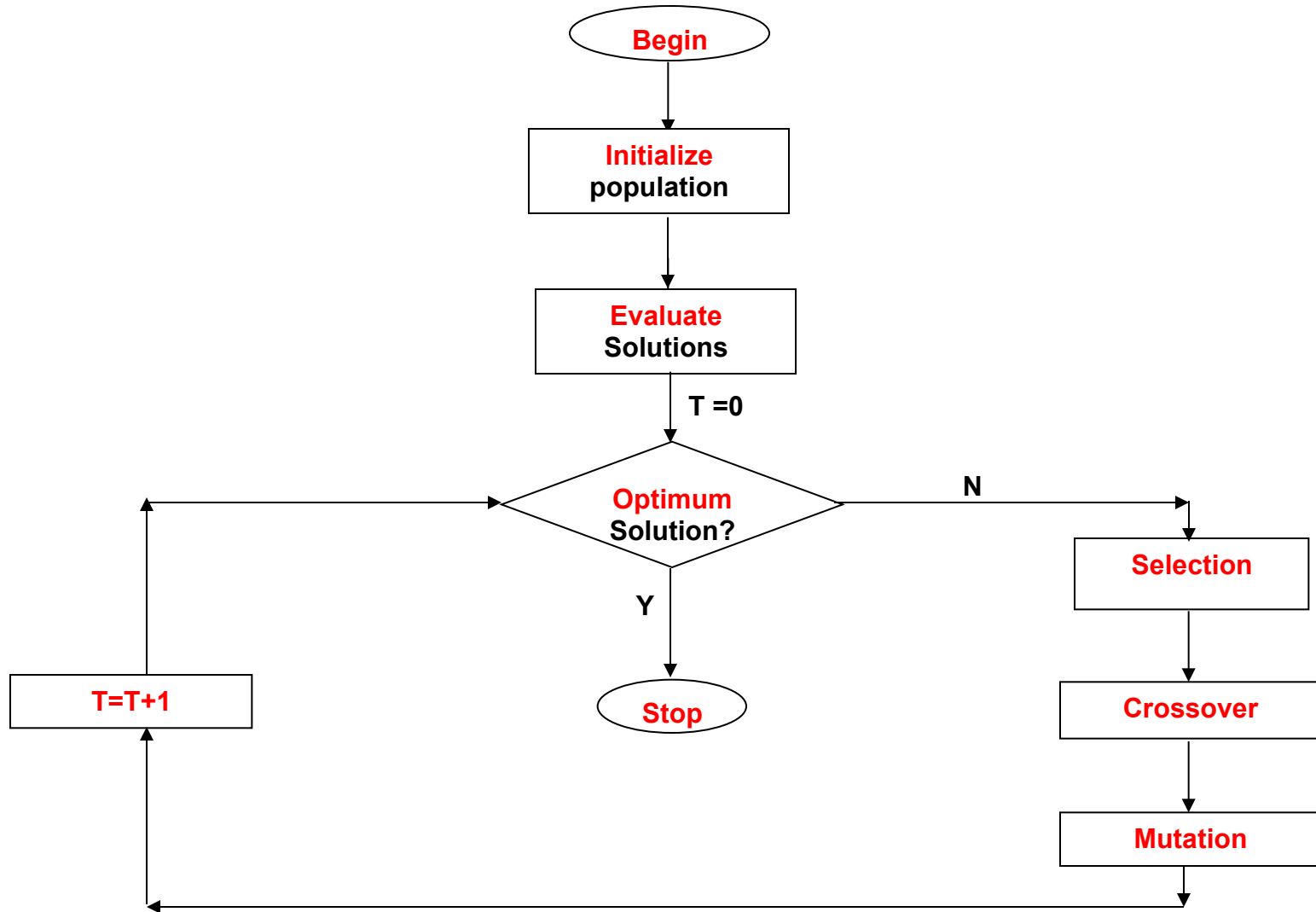
Millions Of Years

Genetic Algorithms implement
Optimization Strategies

by simulating

evolution of species through natural
selection

Working Mechanism Of GA



Simple Genetic Algorithm

```
Simple_Genetic_Algorithm()  
{  
  Initialize the Population;  
  Calculate Fitness Function;  
  
  While(Fitness Value != Optimal Value)  
  {  
    Selection;//Natural Selection, Survival Of  
Fittest  
    Crossover;//Reproduction, Propagate favorable  
characteristics  
  
    Mutation;//Mutation  
    Calculate Fitness Function;  
  }  
}
```

Nature to Computer Mapping

| Nature | Computer |
|---|---|
| Population Individual Fitness Chromosome Gene Reproduction | Set of solutions. Solution to a problem. Quality of a solution. Encoding for a Solution. Part of the encoding of a solution. Crossover |

GA Operators and Parameters

Encoding

ENCODING is a process of representing the solution in the form of a **string** that conveys the **necessary information**

- Just as in a **chromosome**, each **gene controls** a **particular characteristic** of the individual, similarly, **each bit in the string** represents a **characteristic** of the **solution**

Encoding Methods

- **Binary Encoding** – Most common method of encoding.
- **Chromosomes** are strings of **1s and 0s** and each **position** in the **chromosome represents** a particular **characteristic** of the problem.

| | |
|--------------|----------------------|
| Chromosome A | 10110010110011100101 |
| Chromosome B | 11111110000000011111 |

Encoding Methods

- **Permutation Encoding** – Useful in ordering problems such as the **Traveling Salesman Problem (TSP)**
- In **TSP** every chromosome is a string of numbers, each of which represents a city to be visited.

| | |
|--------------|-------------------|
| Chromosome A | 1 5 3 2 6 4 7 9 8 |
| Chromosome B | 8 5 6 7 2 3 1 4 9 |

Encoding Methods

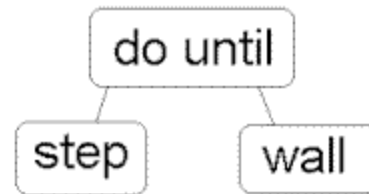
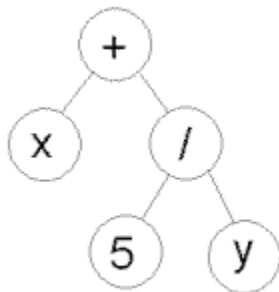
- **Value Encoding** – Used in problems where complicated values, such as **real numbers**, are used and where **binary encoding would not suffice**

Good for some problems, but **often necessary to develop some specific crossover and mutation techniques** for these chromosomes.

| | |
|---------------------|--|
| Chromosome A | 1.235 5.323 0.454 2.321 2.454 |
| Chromosome B | (left), (back), (left), (right), (forward) |

Encoding Methods

- **Tree Encoding** – this encoding is used mainly for evolving programs or expressions, i.e. for Genetic Programming, or Classification
- **Tree Encoding** - every chromosome is a tree of some objects, such as values/arithmetic operators or commands in a programming language or a rule
- $(+x(/5y))$ (do_until step wall)



Genetic Algorithm Operations

Initialization

Selection

Recombination

Reproduction

Termination

Initialization

Individual solutions are randomly generated to form an **initial population**

- Traditionally, the population is generated randomly, covering the entire range of possible solutions (the search space)

Fitness Function

A fitness function quantifies the **optimality** of a solution (chromosome) so that that **particular solution** may be **ranked** against all the other solutions.

- **A fitness value** is assigned to each solution depending on **how close** it actually is to **solving the problem**
- **Ideal fitness function** **correlates closely to goal** and is quickly computable.
- In **TSP**, **$f(x)$** is the **sum of distances** between the cities in solution
- **The lesser** the value, the **fitter** the solution is

Recombination

Recombination is a process that **determines** which solutions are to be **preserved** and allowed to **reproduce** and which ones deserve to **die out**

- **The primary objective** of the **recombination operator** is to **emphasize the good solutions** and **eliminate the bad solutions** in a population, **while keeping the population size constant**
- “Selects The Best, Discards The Rest”
- “Recombination” is different from “Reproduction”

Recombination

- **Identify** the **good solutions** in a population.
- **Make multiple copies** of the good solutions.
- **Eliminate bad solutions** from the population so that **multiple copies** of **good solutions** can be **placed** in the population

Selection Methods

There are many different techniques which a **genetic algorithm** can use to select the individuals to be copied over into the next generation

Listed are some of the most commonly used:

- **Roulette-Wheel Selection**
- **Tournament Selection**
 - **Elitist Selection**
 - **Rank Selection**
- **Hierarchical Selection**

Roulette Wheel Selection

- Each current string in the population has a slot assigned to it which is in proportion to its fitness
- We spin the weighted roulette wheel thus defined n times (where n is the total number of solutions)
- Each time Roulette Wheel stops, the string corresponding to that slot is created

Strings that are fitter are assigned a larger slot and hence have a better chance of appearing in the new population

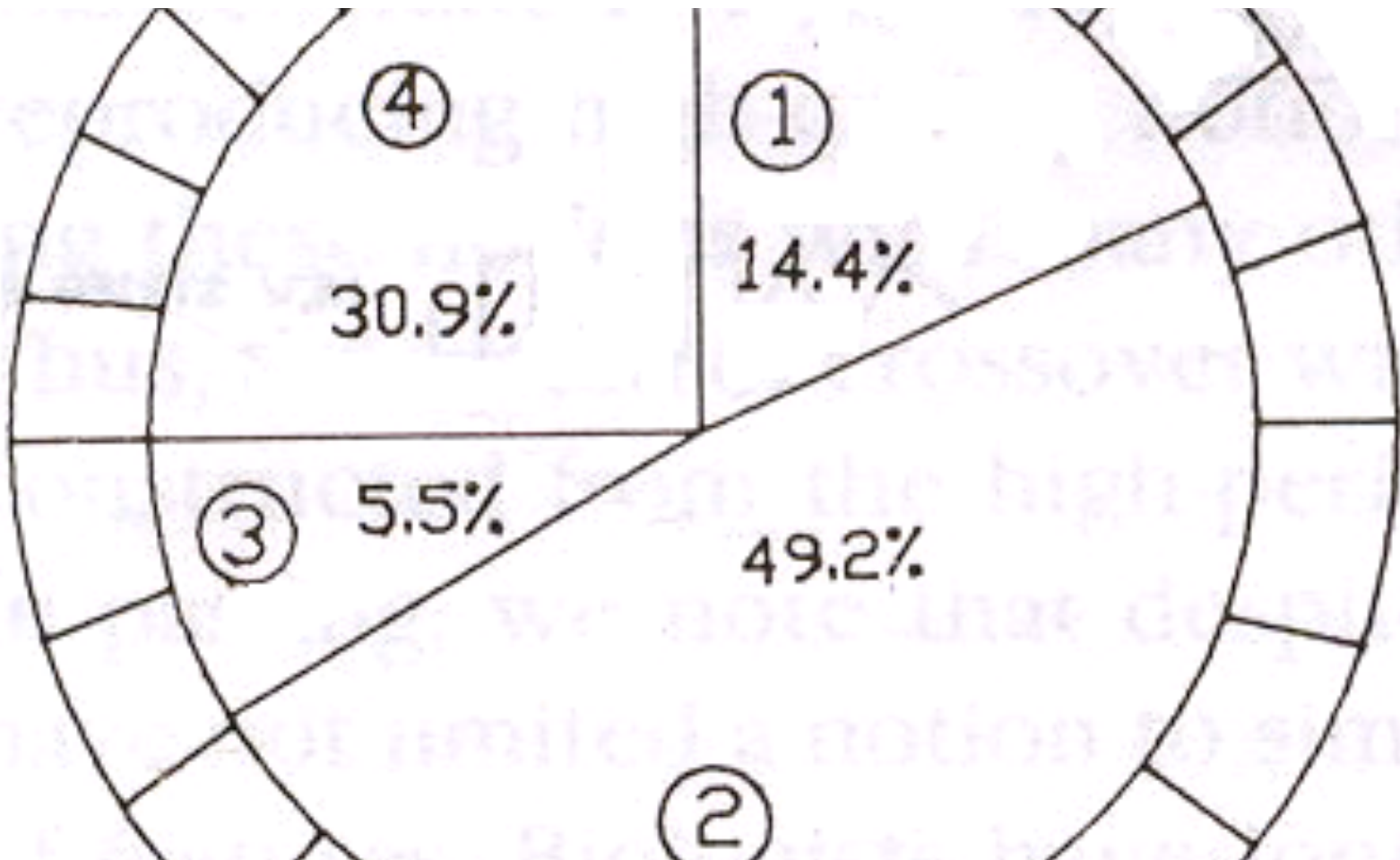
Example Of Roulette Wheel Selection

$$\text{Prob}_i = f(i) / \sum_i f(i)$$

| No. | Initial Population | Fitness f(i) | % of Total |
|---------------------------------------|---------------------------|---------------------|-------------------|
| 1 | 01101 | 169 | 14.4 |
| 2 | 11000 | 576 | 49.2 |
| 3 | 01000 | 64 | 5.5 |
| 4 | 10011 | 361 | 30.9 |
| Total $\sum_i f(i)$ | | 1170 | 100.0 |

Roulette Wheel For Example

We spin 4 times



Tournament Selection

GA runs a "tournament" among a few individuals chosen at random from the **population** and selects the winner (the one with the best fitness) for **crossover**

Two chromosomes are picked out of the pool, their fitness is compared, and **the better** is permitted to **reproduce**

- Deterministic **tournament selection** selects the **best individual** in each tournament
- Independent of **Fitness function**

ADVANTAGE: Decreases computing time, Works on parallel architecture.

Tournament Selection (Pseudo Code)

TS_Procedure_nonDeterministic

{

1. **choose** k (the tournament size) **individuals** from the population **at random**
2. **choose** the **best individual** from pool/tournament **with probability** p
3. **choose** the **second best** individual with **probability** $p*(1-p)$
4. **choose** the **third best** individual with **probability** $p*((1-p)^2)$

and so on...

}

Hierarchical Selection

Individuals go through **multiple rounds** of **selection each generation**

Lower-level evaluations are faster and **less discriminating**

Those that **survive** to **higher levels** are **evaluated more rigorously**

ADVANTAGE: **Efficient usage of computing time** by **weeding out** non-promising **candidate chromosomes**

Rank Selection

Rank selection first **rank**s the population and then every chromosome receives **fitness** from this ranking

Selection is based on this **rank**ing rather than **absolute differences** in **fitness**

The worst will have **fitness 1**, **second worst** will have **fitness 2**, etc...

and **the best** will have **fitness N**

(where **N** is the number of chromosomes in population)

ADVANTAGE: **Preserves genetic diversity** (by preventing dominance of “fitter” chromosomes)

Reproduction

GA Reproduction operators:

Crossover

Mutation

Elitism

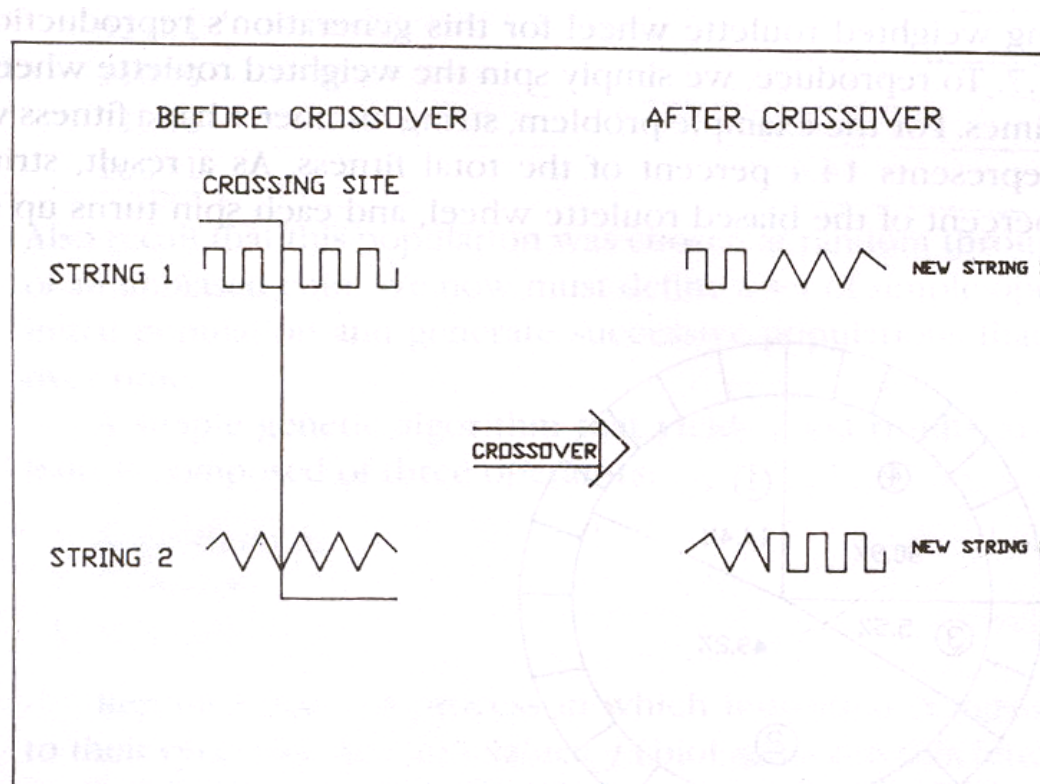
Crossover

Crossover is a **GA operator** in the **process** in which **two chromosomes** (strings) **combine** their **genetic material** (bits) to **produce** a **new offspring** which possesses both their characteristics.

- **Two strings** are picked from the **mating pool** **at random** to **cross over**
- The method chosen for **Crossover** depends on the **Encoding Method**

Crossover Methods

- **Single Point Crossover**- a random point is chosen on the individual chromosomes (strings) and the genetic material is **exchanged at this point**



Crossover Methods

- **Single Point Crossover**

| | |
|---------------------|----------------------------|
| Chromosome1 | 11011 00100110110 |
| Chromosome 2 | 11011 11000011110 |
| Offspring 1 | 11011 11000011110 |
| Offspring 2 | 11011 00100110110 |

Crossover Methods

- **Two-Point Crossover**- two random points are chosen on the individual chromosomes (strings) and the genetic material is **exchanged** at these points

| | |
|---------------------|-------------------------------|
| Chromosome1 | 11011 00100 110110 |
| Chromosome 2 | 10101 11000 011110 |
| Offspring 1 | 10101 00100 011110 |
| Offspring 2 | 11011 11000 110110 |

NOTE: These chromosomes are different from the last example

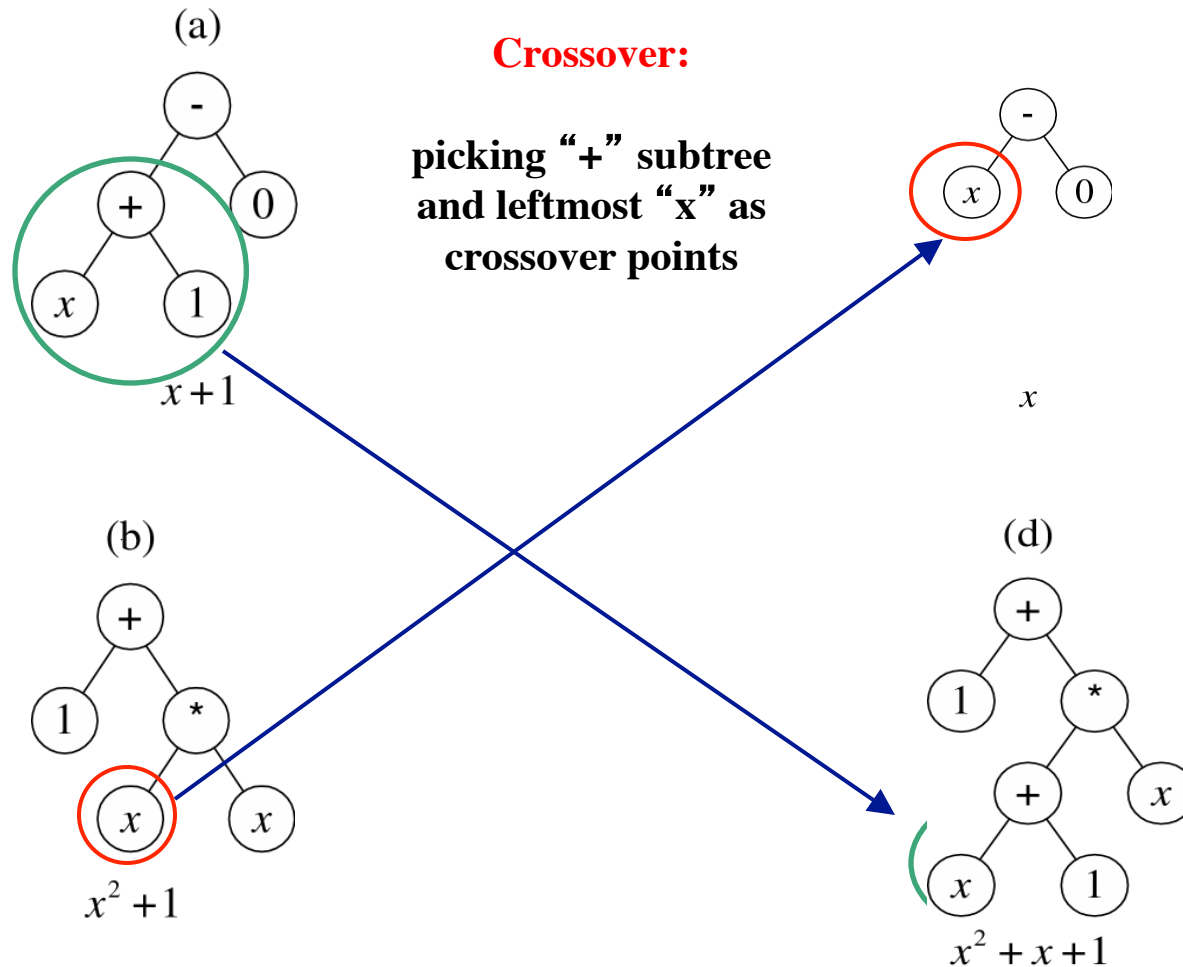
Crossover Methods

- **Uniform Crossover**- each gene (bit) is selected randomly from one of the corresponding genes of the parent chromosomes

| | |
|---------------------|-------------------------------|
| Chromosome1 | 11011 00100 110110 |
| Chromosome 2 | 10101 11000 011110 |
| Offspring | 10111 00000 110110 |

NOTE: Uniform Crossover yields ONLY 1 offspring

Trees Crossover



Crossover

- **Crossover** between **2 good solutions**
- **MAY NOT ALWAYS** yield a **better** or **as good** a solution
- Since parents **are good**, **probability** of the child being **good is high**
- **If offspring** is **not good** (poor solution), it will be **removed** in the next iteration during **Selection**

Elitism

Elitism is a method in which **copies the best chromosome** are **added** to the new offspring population **before crossover** and **mutation**

- When creating a **new population** by **crossover** or **mutation** the **best chromosome** **might be lost**
- **Elitism** lets **GA to retain** some number of the **best individuals** at each generation
- It has been found that **elitism** significantly **improves performance**

Mutation

Mutation is the **process** by which a string is **deliberately changed** so as to maintain **diversity** in the **population set**

We saw in the giraffes' example, that mutations could be beneficial

Mutation Probability- determines how often the parts of a chromosome will be mutated

Mutation

A common method of **implementing** the **mutation operator** involves generating a **random variable** for **each bit** in a sequence

This **random variable** tells a **particular bit** will be **modified**

Example of Mutation

- For chromosomes using **Binary Encoding**, randomly selected bits are **inverted**

| | |
|-------------------|-------------------------------------|
| Offspring | 1101 1 00100 1 1 0110 |
| Mutated Offspring | 1101 0 00100 1 0 0110 |

NOTE: The number of bits to be inverted depends on the **Mutation Probability**

Crossover vs Mutation

Exploration: Discovering promising areas in the search space, i.e. gaining information on the problem

Exploitation: Optimising within a promising area, i.e. using information

There is **co-operation** AND **competition** between them

Crossover is explorative, it makes a **big jump** to an area somewhere **“in between”** **two (parent) areas**

Mutation is exploitative, it creates **random small diversions**, thereby **staying near** (in the area of) **the parent**

Simple Genetic Algorithm (Reproduction Cycle)

Select parents for the mating pool

(size of mating pool = population size)

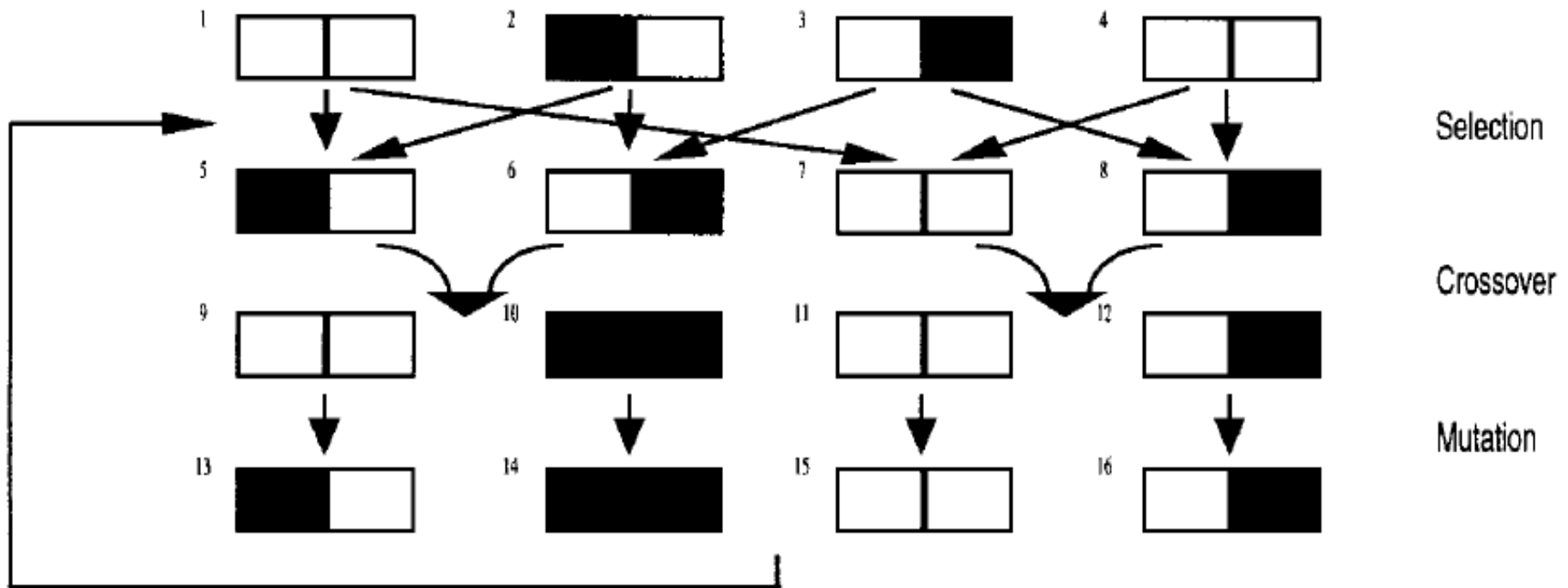
Shuffle the mating pool

For each consecutive pair apply **crossover** with probability P_c ,
otherwise **copy parents**

For each offspring apply **mutation** (bit-flip with probability P_m
independently for each bit)

Replace the whole population with the **resulting offsprings**

One generation of a **genetic algorithm**, consisting of - from top to bottom - **selection, crossover, and mutation stages**



Genetic Algorithms To Solve The Traveling Salesman Problem (TSP)

The Problem

The **Traveling Salesman Problem** is defined as follows:

‘We are given a **set of cities** and a symmetric distance matrix that indicates the cost of travel from each city to every other city.

The goal is to **find the shortest circular tour**, visiting every city **exactly once**, so as to **minimize the total travel cost**, which includes the cost of traveling from the last city **back** to the first city’.

Encoding

- We represent **every city with an integer**

- **Consider** 6 Indian cities –

Mumbai, Nagpur , Calcutta, Delhi , Bangalore and Chennai and assign a number to each.

| | | |
|-----------|---|---|
| Mumbai | 1 | → |
| Nagpur | 2 | → |
| Calcutta | 3 | → |
| Delhi | 4 | → |
| Bangalore | 5 | → |
| Chennai | 6 | → |

Encoding

- Thus a **path** would be represented as a **sequence of integers from 1 to 6**
- **The path [1 2 3 4 5 6]** represents a path from Mumbai to Nagpur, Nagpur to Calcutta, Calcutta to Delhi, Delhi to Bangalore, Bangalore to Chennai, and finally from Chennai to Mumbai
- This is an example of **Permutation Encoding** as the **position of the elements determines the fitness** of the solution.

Fitness Function

- The **fitness function** is be the **total cost of the tour** represented by each chromosome.

The **fitness function** is calculated as the **sum of the distances** traversed in each travel segment

The **Lesser The Sum, The Fitter The Solution**
Represented By That Chromosome

Distance/Cost Matrix For TSP

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|------|------|------|------|------|------|
| 1 | 0 | 863 | 1987 | 1407 | 998 | 1369 |
| 2 | 863 | 0 | 1124 | 1012 | 1049 | 1083 |
| 3 | 1987 | 1124 | 0 | 1461 | 1881 | 1676 |
| 4 | 1407 | 1012 | 1461 | 0 | 2061 | 2095 |
| 5 | 998 | 1049 | 1881 | 2061 | 0 | 331 |
| 6 | 1369 | 1083 | 1676 | 2095 | 331 | 0 |

Cost matrix for six city example.

Distances in Kilometers

Fitness Function

- So, for a chromosome **[4 1 3 2 5 6]**, the total cost of travel or fitness will be calculated as shown below
- **Fitness** = $1407 + 1987 + 1124 + 1049 + 331 + 2095$
= **7993 km**
- Since our **objective** is to **Minimize the distance**, the lesser the total distance, the **fitter the solution**.

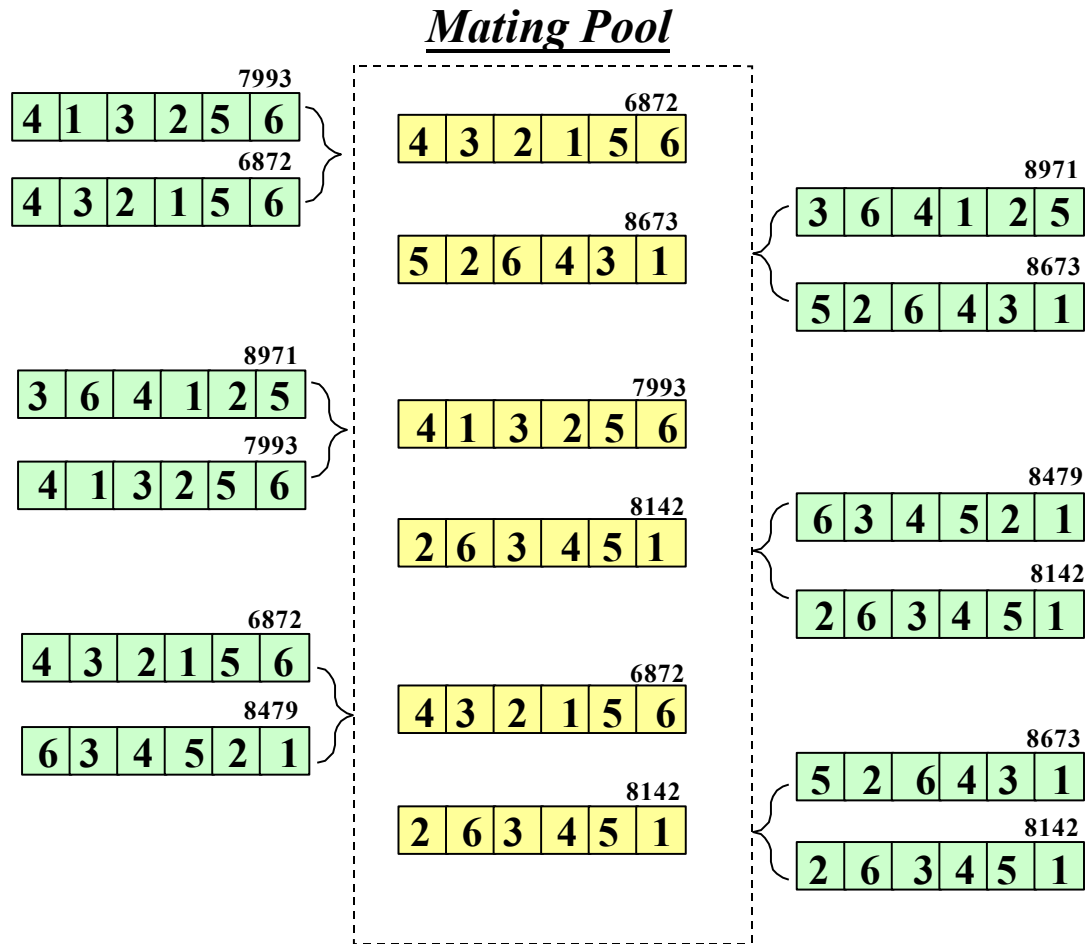
Selection Operator

We use **Tournament Selection**

As the name suggests **tournaments** are played between **two solutions** and the **better solution** is chosen and placed in the **mating pool**

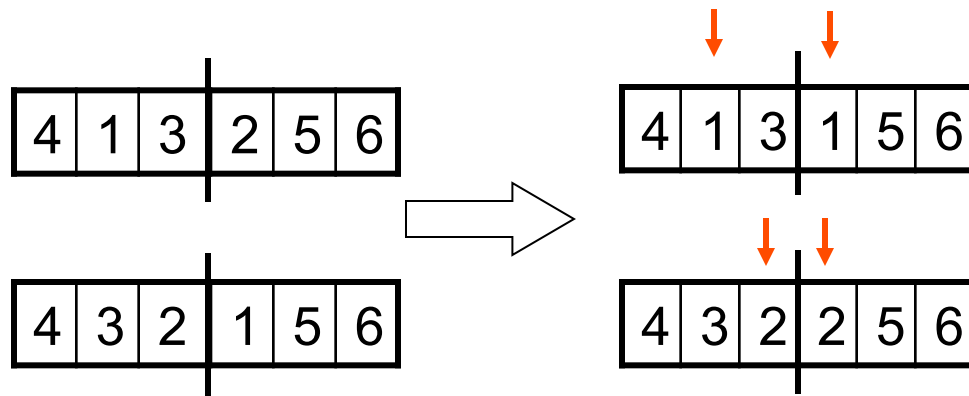
Two other solutions are picked again and another slot in the **mating pool** is filled up with the **better solution**

Tournament



Why we cannot use single-point crossover:

- **Single point crossover** method randomly selects a crossover point in the string and swaps the substrings.
- This may produce some **invalid offsprings** as shown below.



Crossover Operator

- We use the **Enhanced Edge Recombination** operator (T.Starkweather, et al, '*A Comparison of Genetic Sequencing Operators*, International Conference of GAs, 1991)
- This operator is different from other genetic sequencing operators in that it emphasizes **adjacency information** instead of the order or position of items in the sequence.
- The algorithm for the **Edge-Recombination Operator** involves constructing an **Edge Table** first

Edge Table

The **Edge Table** is an *adjacency table* that lists links **into** and **out of** a city found in the **two parent sequences**.

If an item is **already** in the **edge table** and we are trying to insert it again, that element of a sequence must be a **common edge** and is **represented** by **inverting its sign**.

Finding The Edge Table

Parent 1

| | | | | | |
|---|---|---|---|---|---|
| 4 | 1 | 3 | 2 | 5 | 6 |
|---|---|---|---|---|---|

Parent 2

| | | | | | |
|---|---|---|---|---|---|
| 4 | 3 | 2 | 1 | 5 | 6 |
|---|---|---|---|---|---|

| | | | | |
|---|----|----|----|---|
| 1 | 4 | 3 | 2 | 5 |
| 2 | -3 | 5 | 1 | |
| 3 | 1 | -2 | 4 | |
| 4 | -6 | 1 | 3 | |
| 5 | 1 | 2 | -6 | |
| 6 | -5 | -4 | | |

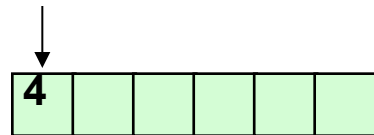
Enhanced Edge Recombination Algorithm

1. Choose the initial city from one of the two parent tours. (It can be chosen randomly as according to criteria outlined in **step 4**). This is the **current city**
2. Remove all occurrences of the **current city** from the left hand side of the edge table. (These can be found by referring to the edge-list for the **current city**)
3. If the **current city** has entries in it's edge-list, go to **step 4** otherwise **go to step 5**
4. Determine which of the cities in the edge-list of the **current city** has the fewest entries in it's own edge-list. The city with fewest entries becomes the **current city**. In case a negative integer is present, it is given preference. Ties are broken randomly. **Go to step 2.**
5. If there are no remaining **unvisited cities**, then *stop*.
6. Otherwise, **randomly choose an unvisited** city and go to **step 2.**

Example Of Enhanced Edge Recombination Operator

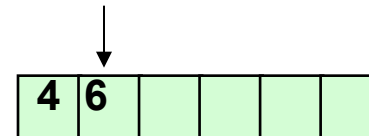
Step 1

| | | | | |
|---|----|----|----|---|
| 1 | 4 | 3 | 2 | 5 |
| 2 | -3 | 5 | 1 | |
| 3 | 1 | -2 | 4 | |
| 4 | -6 | 1 | 3 | |
| 5 | 3 | 2 | -6 | |
| 6 | -5 | -4 | | |



Step 2

| | | | |
|---|----|----|----|
| 1 | 3 | 2 | 5 |
| 2 | -3 | 5 | 1 |
| 3 | 1 | -2 | |
| 4 | -6 | 1 | 3 |
| 5 | 3 | 2 | -6 |
| 6 | -5 | | |



Example Of Enhanced Edge Recombination Operator

Step 3

| | | | |
|---|----|----|---|
| 1 | 3 | 2 | 5 |
| 2 | -3 | 5 | 1 |
| 3 | 1 | -2 | |
| 4 | 1 | 3 | |
| 5 | 3 | 2 | |
| 6 | -5 | | |

↓

| | | | | | |
|---|---|---|--|--|--|
| 4 | 6 | 5 | | | |
|---|---|---|--|--|--|

Step 4

| | | |
|---|----|----|
| 1 | 3 | 2 |
| 2 | -3 | 1 |
| 3 | 1 | -2 |
| 4 | 1 | 3 |
| 5 | 3 | 2 |
| 6 | | |

↓

| | | | | | |
|---|---|---|---|--|--|
| 4 | 6 | 5 | 1 | | |
|---|---|---|---|--|--|

Example Of Enhanced Edge Recombination Operator

Step 5

| | | |
|---|----|---|
| 1 | 3 | 2 |
| 2 | -3 | |
| 3 | -2 | |
| 4 | 3 | |
| 5 | 3 | 2 |
| 6 | | |

↓

| | | | | | |
|---|---|---|---|---|--|
| 4 | 6 | 5 | 1 | 3 | |
|---|---|---|---|---|--|

Step 6

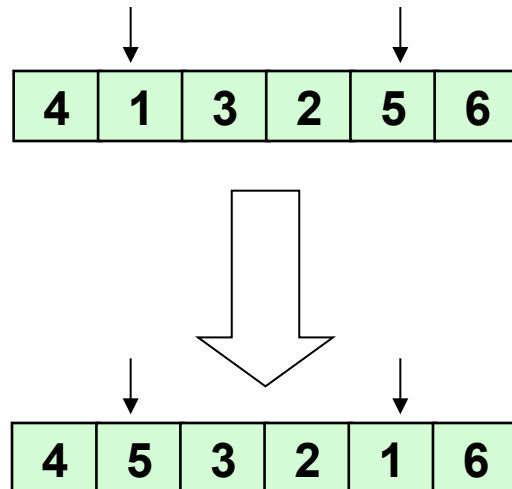
| | |
|---|----|
| 1 | 2 |
| 2 | |
| 3 | -2 |
| 4 | |
| 5 | 2 |
| 6 | |

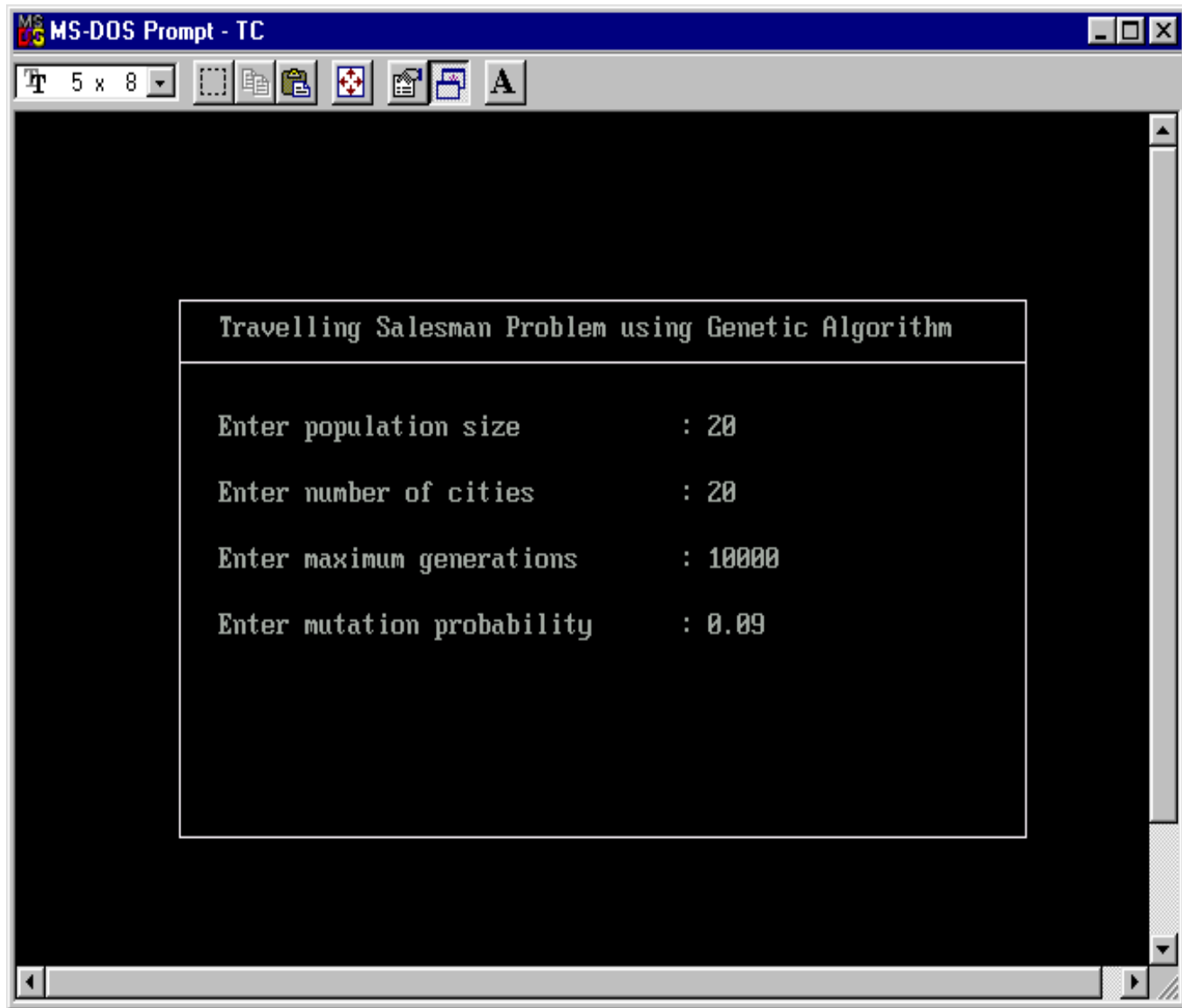
↓

| | | | | | |
|---|---|---|---|---|---|
| 4 | 6 | 5 | 1 | 3 | 2 |
|---|---|---|---|---|---|

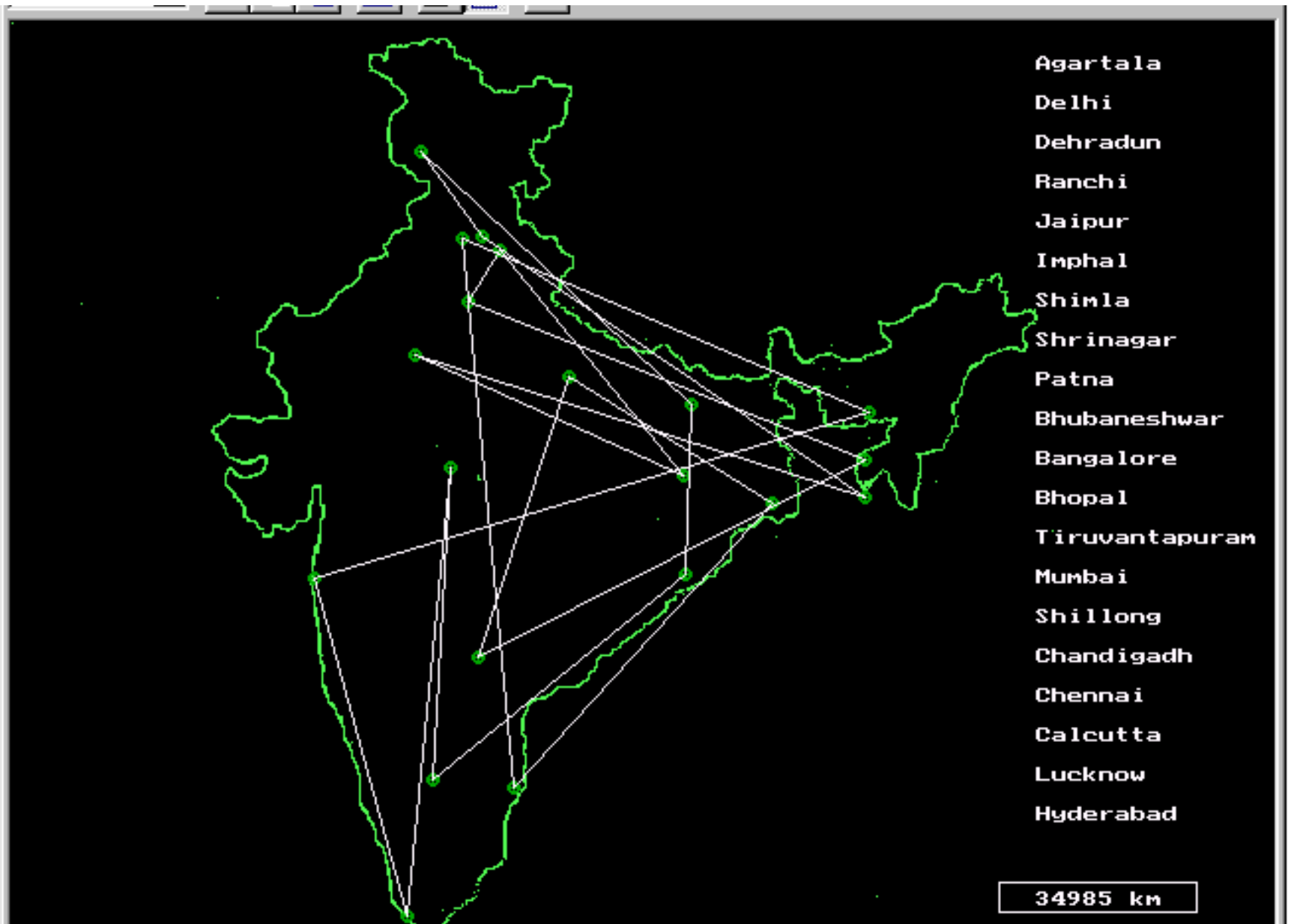
Mutation Operator

- **The mutation operator** induces a change in the solution, so as to maintain diversity in the population and prevent **Premature Convergence**
- We **mutate the string** by **randomly selecting any two cities and interchanging their positions** in the solution, thus giving rise to a new tour.

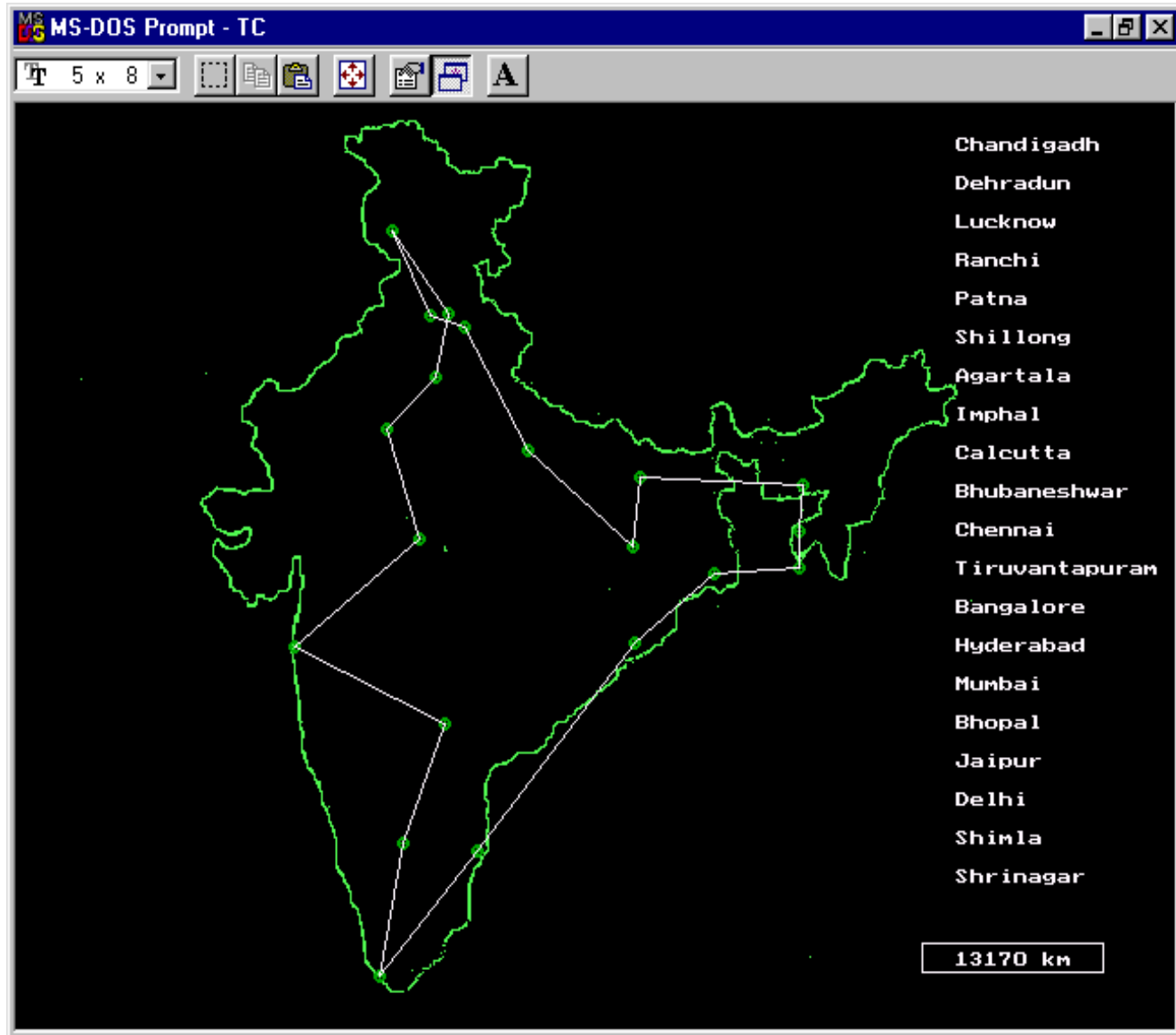




Input To Program



Initial Output For 20 cities : Distance=34985 km
Initial Population



**Final Output For 20 cities : Distance=13170 km
Generation 4786**

Advantages Of GA

GAs search for the function **optimum** starting from a **population of points** of the **function domain**, **not a single one**

This characteristic suggests that **GAs** are **global search methods**

- They can, in fact, **climb many peaks in parallel**, reducing the probability of finding **local minima**, which is one of the **drawbacks** of traditional **optimization methods**

Advantages of GA

GAs only use the **information** about
the objective function

GAs do not require knowledge of any other
auxiliary information

They allow a **number of problems** to be solved without the
need to formulate **restrictive assumptions**

For this reason, **GAs** are often called **blind search methods**

Advantages of GAs

- **GAs** use **probabilistic transition rules**

during iterations, unlike the **traditional methods** that **use fixed transition rules**

This makes them **more robust** and applicable to a **large range of problems**.

Advantages of GAs

- **GAs can be easily implemented on parallel machines**
- Since in real-world design **optimization problems**, most computational time is spent in **evaluating a solution**, with **multiple processors** all **solutions in a population** can be evaluated in a **distributed manner**

This reduces substantially the overall **computational time**

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