Plane Equation

- Given three points, they determine a plane

\[ p_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} \]

\[ p_b = \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} \]

\[ p_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \]

where \( p_a, p_b, \) and \( p_c \) are not co-linear!

- Normal of the plane

\[ n = \frac{(p_c - p_a) \times (p_b - p_a)}{|(p_c - p_a) \times (p_b - p_a)|} \]

\[ n = \begin{bmatrix} A \\ B \\ C \end{bmatrix} \]
- Arbitrary point on the plane
\[
p = \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

- Equation (implicit function)
\[
f(x, y, z) = 0
\]

- Plane equation derivation
\[
(p - p_a) \cdot n = 0
\]
\[
(x - x_a)A + (y - y_a)B + (z - z_a)C = 0
\]
\[
Ax + By + Cz = (Ax_a + By_a + Cz_a) = 0
\]
\[
Ax + By + Cz + D = 0
\]

where
\[
D = -(Ax_a + By_a + Cz_a)
\]

- Explicit expression (if $C$ is non-zero)
\[
z = -\frac{1}{C}(Ax + By + D)
\]
This can be generalized to both $x$ and $y$

- **Parametric representation**

  \[ p(u,v) = p_a + (p_b - p_a)u + (p_c - p_a)v \]

- **Line-Plane intersection**

  \[ l(u) = p_0 + (p_1 - p_0)u \]

  \[
  (n) \times (p_0 + (p_1 - p_0)u) + d = 0
  \]

  \[
  u = -\frac{n \times p_0}{n \times p_1 - n \times p_0} = -\frac{\text{plane}(p_0)}{\text{plane}(p_1) - \text{plane}(p_0)}
  \]

- **Parametric representation**!
Plane and Intersection
Orthographic View Volume

- View-volume plane equations
  - left plane
  - right plane
  - bottom plane
  - top plane
  - front plane
  - back plane

- Assume all the normals point into the view volume

- $x - \text{left} = 0$
- $-x + \text{right} = 0$
- $y - \text{bottom} = 0$
- $-y + \text{top} = 0$
- $z - \text{near} = 0$
\[ z + \text{far} = 0 \]
Perspective View Volume

- Again, six planes!
  - $x + (\text{left} \times z)/(\text{near}) = 0$
  - $-x - (\text{right} \times z)/(\text{near}) = 0$
  - $y + (\text{bottom} \times z)/(\text{near}) = 0$
  - $-y - (\text{top} \times z)/(\text{near}) = 0$
  - $-z - \text{near} = 0$
  - $z + \text{far} = 0$
3D Clipping

- Make use of plane equations

- Determine the sign of the plane equation

- If $plane(p) > 0$, then $p$ is INSIDE!

- Clipping operations
  - point
  - line
  - polygon
  - complicated objects

- Clipping algorithms

- View volume clipping

- 2D algorithms can be generalized to 3D
  - Cohen-Sutherland line-clipping
  - Sutherland-Hodgeman algorithm
View Volume Projection
View Volume Projection