# cse303
# ELEMENTS OF THE THEORY OF COMPUTATION

Professor Anita Wasilewska

# LECTURE 6a

# REVIEW for Q2

**Q2** covers Lecture 5 and Lecture 6

**Chapter 2** - Deterministic Finite Automata DFA

**Chapter 2** - Nondeterministic Finite Automata NDFA

1. Some YES- NO Questions
2. Some Very Short Questions
3. Some Homework Problems

# CHAPTER 2 PART 1
# Deterministic Finite Automata DFA
# Nondeterministic Finite Automata NDFA

## Short YES/NO Questions

**Write your answers** and only after writing them check the solutions

**Q1** Alphabet $\Sigma$ of any deterministic finite automaton M is always non-empty

**Q2** The set $K$ of **states** of any deterministic finite automaton is always non-empty

**Q3** A configuration of a DF Automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$

**Q4** Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a **transition relation** iff he following condition holds
$(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$

## Short YES/NO Questions

**Q5** A configuration of a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$

**Q6** Given $M = (K, \Sigma, \delta, s, F)$ we define

$L(M) = \{w \in \Sigma^* : \ ((s, w) \vdash^*{}_M (q, e))$ for some $q \in K\}$

**Q7** Given $M = (K, \Sigma, \delta, s, F)$ we define

$L(M) = \{w \in \Sigma^* : \ \exists_{q \in K} \ ((s, w) \vdash^*{}_M (q, e))\}$

**Q8** If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic

**Q9** For any automata $M$, we have that $L(M) \neq \emptyset$

**Q10** For any DFA $M = (K, \ \Sigma, \ \delta, \ s, \ F)$,

$e \in L(M)$ if and only if $s \in F$

## Short YES/NO Questions

**Q11** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times \Sigma^* \times K$

**Q12** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$

**Q13** The set of all configurations of any non-deterministic state automata is always non-empty

**Q14** We say that two automata $M_1, M_2$ (deterministic or nondeterministic) are the same , i.e. $M_1 = M_2$ if and only if $L(M_1) = L(M_2)$

**Q15** For any DFA M, there is is a NDFA M', such that $M \approx M'$

## Short YES/NO Questions

**Q16** For any NDFA M, there is is a DFA M', such that $M \approx M'$

**Q17** If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture

**Q18** We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a **computation** of the length $n$ from $(q, w)$ to $(q', w')$ as a **sequence**

$$(q_1, w_1),\ (q_2, w_2), ..., (q_n, w_n), \quad n \geq 1$$

of **configurations**, such that

$q_1 = q,\ q_n = q',\ w_1 = w,\ w_n = w'$ and
$(q_i, w_i) \vdash^*{}_M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$

**Statement:** For any $M$ a computation $(q, w)$ exists

# Short YES/NO Questions

Here are solutions to some short YES/NO Questions for material covered in Chapter 2, Part 1

Solving Quizzes and Tests you have to write a short **solutions** and **circle** the answer

You will get **0 pts** if you only circle your answer without providing a solution, even if it is correct

Here are some questions

**Q1** Alphabet $\Sigma$ of any deterministic finite automaton M is always non-empty

**no** An alphabet $\Sigma$ is, by definition, any **finite set**, hence it can be empty

**Q2** The set $K$ of **states** of any deterministic finite automaton is always non-empty

**yes** $s \in K$

## Short YES/NO Questions

**Q3** A configuration of a DF Automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$

**no** Configuration is any element $(q, w) \in K \times \Sigma^*$

**Q4** Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a **transition relation** iff the following condition holds
$(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$

**no** Proper condition is:
$(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$

**Q5** A configuration of a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$

**yes** by definition

## Short YES/NO Questions

**Q6** Given $M = (K, \Sigma, \delta, s, F)$ we define

$L(M) = \{w \in \Sigma^* : \ ((s, w) \vdash^*_M (q, e))$ for some $q \in K\}$

**no** Must be: for some $q \in F$

**Q7** Given $M = (K, \Sigma, \delta, s, F)$ we define

$L(M) = \{w \in \Sigma^* : \ \exists_{q \in K} \ ((s, w) \vdash^*_M (q, e))\}$

**no** Must be: $\exists_{q \in F} \ ((s, w) \vdash^*_M (q, e))$

**Observe** that **Q7** is really the **Q6** written in symbolic way correctly using the symbol of existential quantifier

## Short YES/NO Questions

**Q8** If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic

**yes** The function $\delta$ is a (special) relation on $K \times \Sigma \times K$, i.e.

$\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K$

**Q9** For any automata $M$, we have that $L(M) \neq \emptyset$

**no** Take M with $\Sigma = \emptyset$ or $F = \emptyset$ then we get $L(M) = \emptyset$

**Q10** For any DFA $M = (K, \Sigma, \delta, s, F)$,

$e \in L(M)$ if and only if $s \in F$

**yes** this is the **DFA Theorem**

# Short YES/NO Questions

**Q11** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when $\Delta \subseteq K \times \Sigma^* \times K$

**no** we must say: $\Delta$ is finite

**Q12** $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$

**yes** this is book definition; do not need to say $\Delta$ is a finite set, as the set $K \times (\Sigma \cup \{e\}) \times K$ is always finite

**Q13** The set of all configurations of any non-deterministic state automata is always non-empty

**yes** the set of all configuration of NDFA is by definition

$K \times \Sigma^* = \{(q, w) : \quad q \in K, \quad w \in \Sigma^*\}$ and we have that

$(s, e) \in K \times \Sigma^*$ even when $\Sigma = \emptyset$ as always $s \in K,\ e \in \Sigma^*$

## Short YES/NO Questions

**Q14** We say that two automata $M_1, M_2$ (deterministic or nondeterministic) are the same , i.e. $M_1 = M_2$ if and only if $L(M_1) = L(M_2)$

**no** we say that $M_1, M_2$ are **equivalent**, i.e. $M_1 \approx M_2$ if and only if $L(M_1) = L(M_2)$

**Q15** For any DFA M, there is is a NDFA M', such that $M \approx M'$

**yes** This is the **Equivalency Theorems 1**

**Q16** For any NDFA M, there is is a DFA M', such that $M \approx M'$

**yes** This is the **Equivalency Theorems 2**

# Short YES/NO Questions

**Q17** If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture

**yes** $\Sigma \cup \{e\} \subseteq \Sigma^*$

**Q18** We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a **computation** of the length $n$ from $(q, w)$ to $(q', w')$ as a **sequence**

$$(q_1, w_1), \ (q_2, w_2), ..., (q_n, w_n), \quad n \geq 1$$

of **configurations**, such that

$q_1 = q, \ q_n = q', \ w_1 = w, \ w_n = w'$ and
$(q_i, w_i) \vdash^*{}_M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$

**Statement:** For any $M$ a computation $(q, w)$ exists

**yes** By definition a computation of length one (case n=1) always exists

## Very Short Questions

For all **short questions** given on Quizzes and Tests you will have to do the following

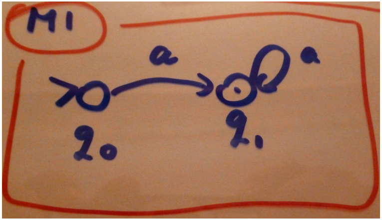
**1.** Decide and explain whether the **diagram** represents a DFA, NDFA or does not

**2.** List all components of $M$ when it represents DFA, NDFA

**3.** Describe $L(M)$ as a **regular expression** when it represents DFA, NDFA

## Very Short Questions

Consider a **diagram** M1




**1.** Yes, it represents a DFA; $\delta$ is a function on $\{q_0, q_1\} \times \{a\}$ and initial state $s = q_0$ exists

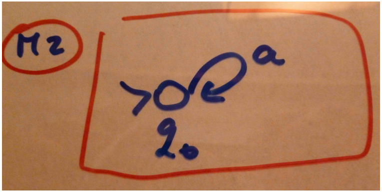
**2.** $K = \{q_0, q_1\}$, $\Sigma = \{a\}$, $s = q_0$, $F = \{q_1\}$,

$\delta(q_0, a) = q_1$, $\delta(q_1, a) = q_1$

**3.** $L(M1) = aa^*$

## Very Short Questions

Consider a **diagram** M2



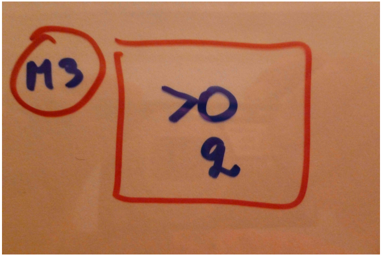

**1.** Yes, it represents a DFA; $\delta$ is a function on $\{q_0\} \times \{a\}$ and initial state $s = q_0$ exists

**2.** $K = \{q_0\},\ \Sigma = \{a\},\ s = q_0,\ F = \emptyset,\ \delta(q_0, a) = q_0$

**3.** $L(M2) = \emptyset$

## Very Short Questions

Consider a **diagram** M3



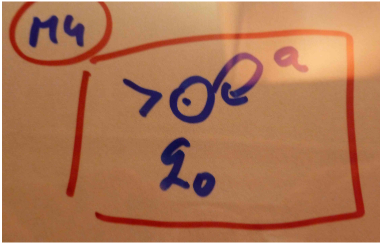

**1.** Yes, it represents a DFA; initial state $s = q_0$ exists

**2.** $K = \{q_0\}$, $\Sigma = \emptyset$, $s = q_0$, $F = \emptyset$, $\delta = \emptyset$

**3.** $L(M3) = \emptyset$

## Very Short Questions

Consider a **diagram** M4




**1.** Yes, it represents a DFA; initial state $s = q_0$ exists

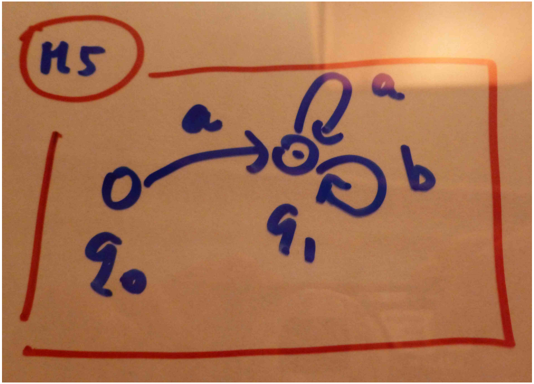
**2.** $K = \{q_0\}, \ \Sigma = \{a\}, \ s = q_0, \ F = \{q_0\}, \ \delta(q_0, a) = q_0$

**3.** $L(M4) = a^*$

**Remark** $e \in L(M4)$ by **DFA Theorem**, as $s = q_0 \in F = \{q_0\}$

## Very Short Questions

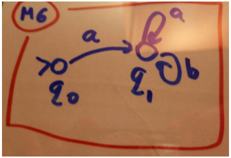
Consider a **diagram** M5




**1.** NO! it is NOT neither DFA nor NDFA - **initial state** does not exist

## Very Short Questions

Consider a **diagram** M6



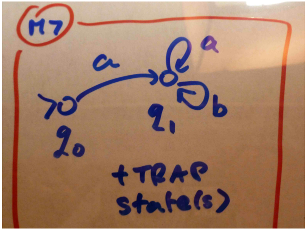

**1.** It is not a DFA; Initial state does exist, but $\delta$ is not a function; $\delta(q_0, b)$ is **not defined** and we didn't say "plus **trap states**"

**2.** It is a NDFA

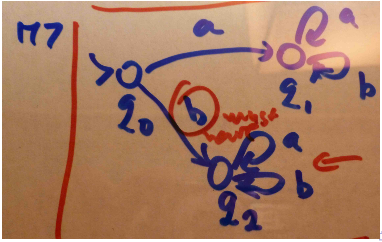
**3.** $L(M6) = \emptyset$

# Very Short Questions

Consider a **diagram** M7

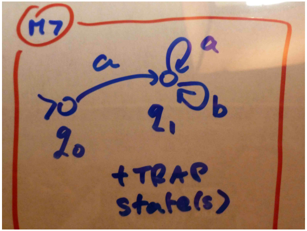



**1.** Yes! it is a DFA with trap states
Initial state exists and we can complete definition of $\delta$ by adding a **trap state** as pictured below



## Very Short Questions

Consider again **diagram** M7




**2.** If we do not say "plus trap states" it represents a NDFA with

$\Delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_1, b, q_1)\}$

**3.** $L(M7) = \emptyset$ as $F = \emptyset$

## Very Short Questions

There is much more Short Questions examples in the section SHORT PROBLEMS at the end of Lecture 5

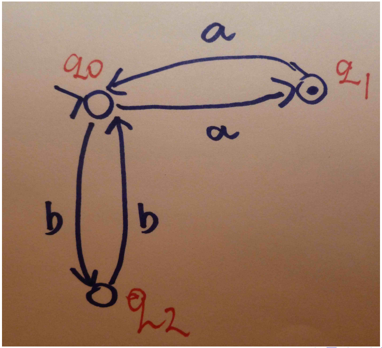
**Problem 1**

Construct deterministic M such that

$$L(M) = \{w \in \Sigma^* : \text{ w has an odd number of a 's and an even number of of b 's }\}$$

**Solution**

Here is the **short diagram** - we must say: plus trap states



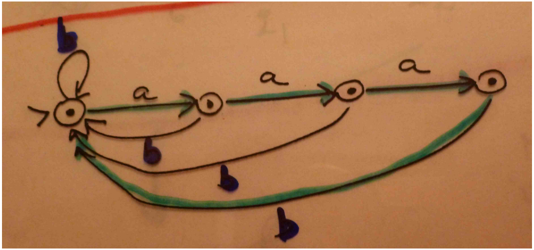
**Problem 2**

Construct a DFA M such that

$L(M) = \{w \in \{a, b\}^* :$ every substring of length 4 in word w

contains at least one b $\}$

**Solution** Here is a **short pattern diagram** (the trap states are not included)

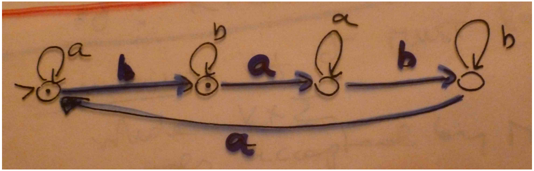


**Problem 3**

Construct a DFA M such that

$$L(M) = \{w \in \{a, b\}^* : \text{ every word } w \text{ contains}$$
$$\text{an even number of sub-strings } ba \}$$

**Solution** Here is a **pattern diagram**




Zero is an even number so we must have that $e \in L(M)$, i.e. we have to make the initial state also a final state

Some Homework Problems

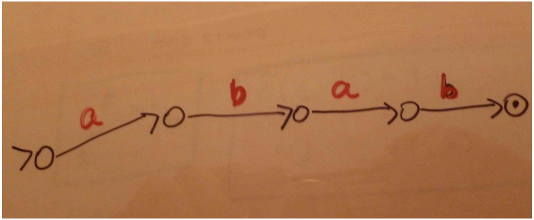
**Problem 4**

Construct a DFA M such that

$$L(M) = \{w \in \{a, b\}^* : \ w \text{ has abab as a substring } \}$$

**Solution** The essential part of the **diagram** must produce abab and it can be surrounded by proper elements on both sides and can be repeated

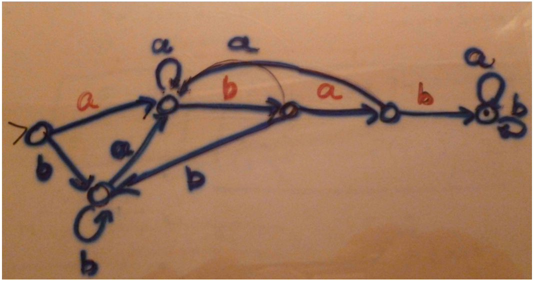
Here is the essential part of the **diagram**



# Problem 4 Solutions

We complete the essential part following the fact that it can be surrounded by proper elements on both sides and can be repeated

Here is the **diagram** of M



Observe that this is a **pattern diagram**; you need to add names of states only if you want to list all components

M does not have trap states

## Some Homework Problems

**Problem 5**

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton $M$, such that

$$L(M) = (ab)^*(ba)^*$$

Specify all components $K, \Sigma, \Delta, s, F$ of $M$ and draw a state diagram

Justify your construction by listing some strings accepted by the state diagram

# Problem 5 Solutions

**Solution 1**

We use the **lecture definition**

Components of $M$ are:

$$\Sigma = \{a, b\}, \ \ K = \{q_0, q_1\}, \ \ s = q_0, \ \ F = \{q_0, \ q_1\}$$

We define $\Delta$ as follows

$$\Delta = \{(q_0, ab, q_0), \ (q_0, e, q_1), \ (q_1, ba, q_1)\}$$

Strings accepted: $ab, \ abab, \ abba, \ babba, ababbaba, ....$

## Problem 5 Solutions

**Solution 2**

We use the **book definition**

Components of $M$ are:

$$\Sigma = \{a, b\}, \quad K = \{q_0, q_1, q_2, q_3\}, \quad s = q_0, \quad F = \{q_2\}$$

We define $\Delta$ as follows

$$\Delta = \{(q_0, a, q_1),\ (q_1, b, q_0),\ (q_0, e, q_2),\ q_2, b, q_3),\ (q_3, a, q_2)\}$$

Strings accepted: $ab,\ abab,\ abba,\ babba, ababbaba, ....$

## Some Homework Problems

**Problem 6**

Let $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0, q_1, q_2, q_3, \}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and

$$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$$

Find the regular expression describing the $L(M)$.

Simplify it as much as you can. Explain your steps

**Solution**

$L(M) = (abc)^* abbb \cup abbb \cup (abc)^* baa \cup ba =$
$(abc)^* abbb \cup (abc)^* baa(abc)^*(abbb \cup baa)$

We used the property: $LL_1 \cup LL2 = L(L_1 \cup L_2)$

**Problem 7**

Let $M = (K, \Sigma, s, \Delta, F)$ for $K = \{q_0, q_1, q_2, q_3, \}$, $s = q_0$, $\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$

**Write down** (you can draw the diagram) an automaton $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

**Solution**

We apply the "stretching" technique to $M$ and the new $M'$ is is as follows.

$M' = (K \cup \{p_1, p_2, ..., p_5\} \Sigma, s = q_0, \Delta', F' = F)$

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$

## Some Homework Problems

I will NOT include this problem on **Q2**, but you have to know how to solve similar problems for **Midterm**

**Problem 8**

Let $M$ be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2\},\ s = q_0, \Sigma = \{a, b\},\ F = \{q_0, q_2\}$ and

$$\Delta = \{(q_0, a, q_1),\ (q_1, b, q_2),\ (q_1, b, q_0),\ (q_2, a, q_0)\}$$

**Write 4 steps** of the general method of transformation a NDFA $M$ , into an **equivalent** $M'$, which is a DFA

## Problem 8

**Reminder**

$E(q) = \{p \in K : (q, e) \vdash^*{}_M (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup_{p \in K} \{E(p) : \exists_{q \in Q} \, (q, \sigma, p) \in \Delta\}$$

**Step 1**

Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$

**Step i+1]**

Evaluate $\delta$ on all states that result from **Step i**

## Problem 8

### Solution

$$\delta(Q,\sigma) = \bigcup_{p\in K}\{E(p) : \exists_{q\in Q}\ (q,\sigma,p) \in \Delta\}$$

### Step 1

$$E(q_0) = \{q_0\},\ E(q_1) = \{q_1\},\ E(q_2) = \{q_2\}$$

$$\delta(\{q_0\},a) = E(q_1) = \{q_1\}, \quad \delta(\{q_0\},b) = \emptyset\}$$

### Step 2

$\delta(\emptyset,a) = \emptyset,\ \ \delta(\emptyset,a) = \emptyset,\ \ \delta(\{q_1\},a) = \emptyset,$

$\delta(\{q_1\},b) = E(q_0) \cup E(q_2) = \{q_0,q_2\} \in F'$

# Problem 8

## Solution

$$\delta(Q,\sigma) = \bigcup_{p\in K} \{E(p) :\ \exists_{q\in Q}\ (q,\sigma,p) \in \Delta\}$$

## Step 3

$$\delta(\{q_0,q_2\},a) = E(q_1) \cup E(q_0) = \{q_0,q_1\}, \quad \delta(\{q_0,q_2\},b) = \emptyset$$

## Step 4

$$\delta(\{q_0,q_1\},a) = \emptyset\ \cup E(q_1) = \{q_1\},$$
$$\delta(\{q_0,q_1\},b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0,q_2\} \in F'$$