cse303
ELEMENTS OF THE THEORY OF COMPUTATION

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LECTURE 14

SMALL REVIEW FOR FINAL

SOME Y/N QUESTIONS

Q1 Given $\Sigma = \emptyset$, there is $L \neq \emptyset$ over Σ

Yes: $\emptyset^* = \{e\}$ and $L = \{e\} \subseteq \Sigma^*$

Q2 There are uncountably many languages over $\Sigma = \{a\}$

Yes: $|\{a\}^*| = \aleph_0$ and $|2^{\{a\}^*}| = \mathcal{C}$ and any set of cardinality $\mathcal{C}$ is uncountable

Q3 Let RE be a set of regular expressions.

$L \subseteq \Sigma^*$ is regular iff $L = L(r)$, for some $r \in RE$

Yes: this is definition of regular language

Q4 $L^* = \{w \in \Sigma^* : \exists_{q \in F}(s, w) \vdash_M^* (q, e)\}$

No: this is definition of $L(M)$, not of L^*

SOME Y/N QUESTIONS

Q5 $L^* = L^+ - \{e\}$

No: only when $e \notin L$

Q6 $L^* = \{w_1 \ldots w_n : \ w_i \in L, \ i = 1, \ldots, n\}$

No: only when $i = 0, 1, \ldots, n$

Q7 For any languages $L_1, L_2 \subseteq \Sigma^*$,

if $L_1 \subseteq L_2$, then $(L_1 \cup L_2)^* = L_2^*$

Yes languages are sets, so $(L_1 \cup L_2) = L_2^*$ when $L_1 \subseteq L_2$

Q8 $((\phi^* \cap a) \cup (\phi \cup b^*)) \cap \phi^*$ represents a language $L = \{e\}$

Yes $((\{e\} \cap \{a\}) \cup \{b\}^*) \cap \{e\} = \{b\}^* \cap \{e\} = \{e\}$

SOME Y/N QUESTIONS

Q9 $L(M) = \{w \in \Sigma^* : \ (q, w) \vdash_M^* (s, e)\}$

No: only when $q \in F$

Q10 $L(M_1) = L(M_2)$ iff M_1 and M_2 are finite automata

No: take as M_1, M_2 any finite automata such that $L(M_1) \neq \phi$ and M_2 such that $L(M_2) = \phi$

Q11 Any finite language is Context Free

Yes: any finite language is regular and we proved that $RL \subset CFL$

Q12 Intersection of any two regular languages is CF language

Yes: Regular languages are **closed under intersection** and $RL \subset CFL$

SOME Y/N QUESTIONS

Q13 Union of a regular and a CF language is a CF language

Yes: $RL \subseteq CFL$ and FCL are closed under union

Q14 If L is regular, there is a PDA M such that $L = L(M)$

Yes: FA is a PDA operating on an empty stock

Q15 $L = \{a^n b^n c^n : n \geq 0\}$ is CF

No: L is not CF, as proved by Pumping Lemma for CF languages

Q16 Let $\Sigma = \{a\}$, then for any $w \in \Sigma^*$ we have that $w^R = w$

Yes: $a^R = a$ and hence $w^R = w$ for $w \in \{a\}^*$

SOME Y/N QUESTIONS

Q17 $A \to Ax, A \in V,\ x \in \Sigma^*$ is the only rule allowed in a regular grammar

No: not only, $A \to xB$ for $B \neq A$ is also a rule of a regular grammar

Q18 Let $G = (\{S, (,)\}, \{(,)\}, R, S)$ for $R = \{S \to SS \mid (S)\}$

$L(G)$ is regular

Yes: $L(G) = \emptyset$ and hence regular

Q19 The grammar with rules $S \to AB, B \to b \mid bB, A \to e \mid aAb$ generates a language $L = \{a^k b^j : k < j\}$

Yes: the rule $A \to e \mid aAb$ produces the same amount of a's and b's, and the rule $B \to bB$ adds only b's

SOME Y/N QUESTIONS

Q20 We can always show that L is regular using Pumping Lemma

No: we use Pumping Lemma to prove (if possible) that L is not regular

Q21 $((p, e, \beta), (q, \gamma)) \in \Delta$ means: read nothing, move from p to q

No: must add: and replace β by γ on the top of the stack

Q22 $L = \{a^n b^m c^n : \; n, m \in N\}$ is CF

Yes: L = L(G) for G with rules $S \rightarrow aSc|B, \; B \rightarrow bB|e$

Q23 Every subset of a regular language is a regular language

No: $L = \{a^n b^n : n \geq 0\} \subseteq a^* b^*$ and L is not regular

SOME Y/N QUESTIONS

Q24 Class of context-free languages is closed under intersection

No: $L_1 = \{a^n b^n c^m : \ n, m \geq 0\}$ is CF,
$L_1 = \{a^m b^n c^n : \ n, m \geq 0\}$ is CF, but
$L_1 \cap L_2 = \{a^n b^n c^n, n \geq 0\}$ is not CF

Q25 A regular language is a CF language

Yes: Regular grammar is a special case of a context-free grammar

Q26 Any regular language is accepted by some PD automaton

Yes: Any regular language is accepted by a finite automaton, and a finite automaton is a PD automaton (that never operates on the stock)

SOME Y/N QUESTIONS

Q27 Turing Machines can read and write

Yes: by definition

Q28 A configuration of a Turing machine $M = (K, \Sigma, \delta, s, H)$ is any element of a set $K \times \Sigma^* \times (\Sigma^*(\Sigma - \{\sqcup\}) \cup \{e\})$, where $\sqcup$ denotes a blank symbol

No: a configuration is an element of a set $K \times \triangleright\Sigma^* \times (\Sigma^*(\Sigma - \{\sqcup\}) \cup \{e\})$

Q29 A computation of a Turing machine can start at any position of $w \in \Sigma$

Yes: by definition

SOME Y/N QUESTIONS

Q30 In Turing machines, words $w \in \Sigma^*$ **can't** contain blank symbols

No: Σ in Turing machine contains the blank symbol $\sqcup$

Q31 It is proved that everything computable (algorithm) is computable by a Turing Machine and vice versa

No: this is Church - Turing Hypothesis, not a theorem

Q32 A Turing machine M **decides** a language $L \subseteq \Sigma^*$, if for any word $w \in \Sigma^*$ the following is true.

If $w \in L$, then then M **accepts** w;

and if $w \notin L$ then M **rejects** w

No: must say: any word $w \in \Sigma_0{}^*$, and $L \subseteq \Sigma_0{}^*$ for $\Sigma_0 = \Sigma - \{\sqcup\}$

SOME PROBLEMS

P1

Let Σ be any alphabet, L_1, L_2 be two languages such that $e \in L_1$ and $e \in L_2$. **Show that**

$$(L_1\Sigma^\star L_2)^\star = \Sigma^\star$$

Solution

By definition, $L_1 \subseteq \Sigma^\star, L_2 \subseteq \Sigma^\star$ and $\Sigma^\star \subseteq \Sigma^\star$. Hence

$$(L_1\Sigma^\star L_2)^\star \subseteq \Sigma^\star$$

We have to show that also

$$\Sigma^\star \subseteq (L_1\Sigma^\star L_2)^\star$$

Let $w \in \Sigma^\star$. We have that also $w \in (L_1\Sigma^\star L_2)^\star$ because$w = ewe$ and $e \in L_1$ and $e \in L_2$

P2

Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton M, such that

$$L(M) = (ab)^*(ba)^*$$

Draw a state diagram. Do not specify all components.
Justify your construction by listing some strings accepted by the state diagram

Solution 1: We use the **lecture definition**

Components of M are:

$\Sigma = \{a, b\},\ K = \{q_0, q_1\},\ s = q_0,\ F = \{q_0, q_1\},$

$$\Delta = \{(q_0, ab, q_0), (q_0, e, q_1), (q_1, ba, q_1)\}$$

You must **draw the diagram** only!

Strings accepted are: $ab, abab, abba, ababba, ...$

You must **trace the computations** accepting these strings!

SOME PROBLEMS

P2

Solution 2: We use the **book definition**

Components of M are:

$\Sigma = \{a, b\}, \ K = \{q_0, q_1, q_2, q_3\}, \ s = q_0, \ F = \{q_2\},$

$$\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}$$

You must **draw the diagram** only!

Strings accepted are: $ab, abab, abba, ababba, ...$

You must **trace the computations** accepting these strings!

SOME PROBLEMS

P3

1. DRAW a DIAGRAM of a PDA M, such that

$$L(M) = \{b^n a^{2n} : n \geq 0\}$$

Solution 1

Here are the components- **you must draw a diagram**!

$M = (K, \Sigma, \Gamma, \Delta, s, F)$ for

$$K = \{s, f\}, \Sigma = \{a, b\}, \Gamma = \{a\}, s, F = \{f\},$$

$$\Delta = \{((s, b, e), (s, aa)), ((s, e, e), (f, e)), ((f, a, a), (f, e))\}$$

SOME PROBLEMS

P3

2. Explain the construction. Write motivation.

Solution

M operates as follows:

Δ **pushes** aa on the top of the stock while M is reading b, **switches** to f (final state) non-deterministically;

and **pops** a while reading a (all in final state)

M puts on the stock two a's for each b, and then **remove** all a's from the stock **comparing** them with a's in the word while in the final state

P3

3. Trace a transitions of M that leads to the acceptance of the string $bbaaaa$

The accepting computation is:

$$(s, bbaaaa, e) \vdash_M (s, baaaa, aa) \vdash_M (s, aaaa, aaaa)$$

$$\vdash_M (f, aaaa, aaaa) \vdash_M (f, aaa, aaa) \vdash_M (f, aa, aa)$$

$$\vdash_M (f, a, a) \vdash_M (f, e, e)$$

Solution 2

$M = (K, \Sigma, \Gamma, \Delta, s, F)$ for

$$K = \{s, f\}, \Sigma = \{a, b\}, \Gamma = \{b\}, s, F = \{f\},$$

$$\Delta = \{((s, b, e), (s, b)), ((s, e, e), (f, e)), ((f, aa, b), (f, e))\}$$

SOME PROBLEMS

P4

Given a **Regular grammar** $G = (V, \Sigma, R, S)$, where

$$V = \{a, b, S, A\}, \ \ \Sigma = \{a, b\},$$

$$R = \{S \to aS \mid A \mid e, \ \ A \to abA \mid a \mid b\}$$

1. Use the construction in the proof of **L-GTheorem:**

Language L is regular if and only if there exists a regular grammar G such that L = L(G)

to construct a **finite automaton** M , such that $L(G) = L(M)$

Draw a **diagram** of M

P4

Solution

Given $R = \{S \to aS \mid A \mid e, \;\; A \to abA \mid a \mid b\}$

we construct a **non-deterministic** finite automata

$$M = (K, \Sigma, \Delta, s, F)$$

as follows:

$$K = (V - \Sigma) \cup \{f\}, \; \Sigma = \Sigma, s = S, \;\; F = \{f\},$$

$$\Delta = \{(S, a, S), (S, e, A), (S, e, f), (A, ab, A), (A, a, f), (A, b, f)\}$$

P4

2. **Trace** a transitions of M that lead to the acceptance of the string $aaaababa$, and **compare** with a derivation of the same string in G

Solution

The accepting **computation** is:

$$(S, aaaababa) \vdash_M (S, aaababa) \vdash_M (S, aababa) \vdash_M (S, ababa)$$

$$\vdash_M (A, ababa) \vdash_M (A, aba) \vdash_M (A, a) \vdash_M (f, e)$$

G **derivation** is:

$$S \Rightarrow aS \Rightarrow aaS \Rightarrow aaaS \Rightarrow aaaA \Rightarrow aaaabA$$

$$\Rightarrow aaaababA \Rightarrow aaaababa$$

SOME PROBLEMS

P5

Prove that the Class of context-free languages is NOT closed under intersection

Proof

Assume that the context-free languages are **are closed** under **intersection**

Observe that both languages

$$L_1 = \{a^n b^n c^m : \ m, n \geq 0\} \text{ and } L_2 = \{a^m b^n c^n : \quad m, n \geq 0\}$$

are **context-free**

So the language $L_1 \cap L_2$ must be **context-free**, but

$$L_1 \cap L_2 = \{a^n b^n c^n : \quad n \geq 0\}$$

and we have proved that $L = \{a^n b^n c^n : \quad n \geq 0\}$ is **not** context-free

Contradiction