CSE303 Q3 PRACTICE SOLUTIONS Spring 2011

YES/NO questions Circle the correct answer. Write SHORT justification.

1. For any finite language L there is a deterministic automata M, such that $L = L(M)$.
   Justify: Any finite language is regular **y**
2. Any regular language is finite.
   Justify: $L = a^*$ is infinite **n**
3. Any finite language is regular.
   Justify: $L = \bigcup\{L_w : w \in L\}$, each L_w is regular and regular languages are closed under finite union. **y**
4. Given L_1, L_2 regular languages over Σ, then $(L_1 \cup (\Sigma^* - L_1))L_2$ is regular.
   Justify: closure of regular languages over union and complement **y**
5. For any M, $L(M) = \bigcup\{R(1, j, n) : \ q_j \in F\}$, where $R(1, j, n)$ is the set of all strings in Σ^* that may drive M from state initial state to state q_j without passing through any intermediate state numbered $n + 1$ or greater, where n is the number of states of M.
   Justify: only when M is a finite automaton **n**
6. The Generalized Finite Automaton accepts regular expressions.
   Justify: accepts regular expressions **y**
7. There is an algorithm that for any finite automata M computes a regular expression r, such that $L(M) = r$ (short hand notation).
   Justify: defined in the proof of Main Theorem **y**
8. Pumping Lemma says that we can always prove that a language is regular.
   Justify: it gives certain characterization of infinite regular languages **n**
9. Pumping Lemma proves that a language is not regular.
   Justify: PL is usually used to prove that an infinite language is not regular **n**
10. $L = \{a^n : n \geq 0\}$ is not regular.
    Justify: $L = a^*$ **n**
11. $L = \{b^n a^n : n \geq 0\}$ is not regular.
    Justify:proved using Pumping Lemma **y**
12. $L = \{a^{2n} : n \geq 0\}$ is regular.
    Justify: $L = (aa)^*$ **y**

13. Let L be a regular language, and $L_1 \subseteq L$, then L_1 is regular.
Justify: $L_1 = \{b^n a^n : n \geq 0\} \subseteq L = b^* a^*$ and L is regular, and L_1 is not regular **n**

14. Let L be a regular language. The language $L^R = \{w^R : \ w \in L\}$ is regular.
Justify: L^R is accepted by a finite automata $M^R = (K \cup s', \Sigma, \Delta', s', F = \{s\})$, where K is the set of states of M accepting L, $s' \notin K$, s the initial state of M, F is the set of final states of M and

$$\Delta' = \{(r, \sigma, p) : (p, \sigma, r) \in \Delta\} \cup \{(s', e, q) : q \in F\},$$

where Δ is the set of transitions of M. **y**

QUESTION 1 Give a direct construction for the closure under intersection of the languages accepted by finite automata. Which of the two constructions, the one given in the textbook or one suggested in this problem, is more efficient when the two languages are given in terms of nondeterministic automata?

Solution

Case 1: deterministic Let

$$M_1 = (K_1, \Sigma, \delta_1, s_1, F_1), \quad M_2 = (K_2, \Sigma, \delta_2, s_2, F_2)$$

be two deterministic automata. We construct

$$M = (K, \Sigma, \delta, s, F),$$

such that

$$L(M) = L(M_1) \cap L(M_2)$$

as follows.

$$K = K_1 \times K_2, \quad s = (s_1, s_2), \quad F = F_1 \times F_2,$$

$$\delta((q_1, q_2), \sigma) = (\delta_1(q_1, \sigma), \delta_2(q_2, \sigma)) = (p_1, p_2).$$

Obviously

$w \in L(M)$ iff $((s_1, s_2), w) \overset{*,M}{\vdash} ((f_1, f_2), e))$ and $f_1 \in F_1, f_2 \in F_2$ iff $(s_1, w) \overset{*,M}{\vdash} (f_1, e))$ for $f_1 \in F_1$ and $(s_2, w) \overset{*,M}{\vdash} (f_2, e))$ for $f_2 \in F_2$ iff $w \in L(M_1)$ and $w \in L(M_2)$ iff $w \in L(M_1) \cap L(M_2)$. We denote

$$M = M_1 \cap M_2.$$

Case 2: nondeterministic Let

$$M_1 = (K_1, \Sigma, \Delta_1, s_1, F_1), \quad M_2 = (K_2, \Sigma, \Delta_2, s_2, F_2)$$

be two nondeterministic automata. We construct

$$M = (K, \Sigma, \delta, s, F),$$

such that

$$L(M) = L(M_1) \cap L(M_2),$$

and denoted by $M = M_1 \cap M_2$ as follows.

$$K = K_1 \times K_2, \quad s = (s_1, s_2), \quad F = F_1 \times F_2,$$

$$\Delta = \{((q_1, q_2), \sigma, (p_1, p_2)) : \ (q_1, \sigma, p_1) \in \Delta_1 \ and \ (q_2, \sigma, p_2) \in \Delta_2 \ and \ \sigma \in \Sigma$$

$$or \ (q_1, e, p_1) \in \Delta_1 \ and \ q_2 = p_2, \ i.e. \ (q_2, \sigma, q_2) \in \Delta_2$$

$$or \ \ (q_2, e, p_2) \in \Delta_2 \ and \ q_1 = p_1 i.e. \ (q_1, \sigma, q_1) \in \Delta_1\}$$

This is called a DIRECT construction of $M = M_1 \cap M_2$, as opposed to the in the proof of the theorem of closure of regular languages under set intersection.

Observe that if M_1, M_2 have each at most n states, our direct construction of produces $M = M_1 \cap M_2$ with at most n^2 states. The construction from the proof of the theorem might generate M with up to $2^{2^{n+1}+1}$ states.

QUESTION 2 Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata M accepting

$$L = \mathcal{L} = a^*(ab \cup ba \cup \emptyset^*)b^*$$

You can just draw a diagrams.

1. Define $M1, M2, M3$ such that $L(M1) = ab, L(M2) = ba, L(M3) = \emptyset^*$

Solution

M1 components:

$$K_1 = \{q_1, q_2\}, s = q_1, F_1 = \{q_2\}, \Delta_1 = \{(q_1, ab, q_2)\}$$

M2 components:

$$K_2 = \{q_3, q_4\}, s = q_3, F_2 = \{q_4\}, \Delta_2 = \{(q_3, ba, q_4)\}$$

M3 components:

$$K_3 = \{q_5, q_6\}, s = q_5, F_3 = \{q_6\}, \Delta_3 = \{(q_5, e, q_6)\}$$

2. Define $M4$ such that $L(M4) = L(M1) \cup L(M2) \cup L(M3)$

Solution

M4 components:

$$K_4 = K_1 \cup K_2 \cup K_3 \cup \{q_7\},\ \ s = q_7, F_7 = F_1 \cup F_2 \cup F_4,$$

$$\Delta_3 = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \{(q_7, e, q_1), (q_7, e, q_3), (q_7, e, q_5)\}$$

3. Define $M5, M6$ such that $L(M5) = a^*, L(M6) = b^*$.

Solution

M5 components:

$$K_5 = \{q_8\}, s = q_8, F_5 = \{q_8\}, \Delta_5 = \{(q_8, a, q_8)\}$$

M6 components:

$$K_6 = \{q_9\}, s = q_, F_6 = \{q_9\}, \Delta_6 = \{(q_9, b, q_9)\}$$

4. Use $M_1 - M_6$ to define M such that $L = L(M)$.

Solution

M components:

$$K = K_4 \cup K_5 \cup K_6 = \{q_1, ...q_0\}, s = q_8, F = \{q_9\},$$

$$\Delta = \bigcup_{i=1...6} \Delta_i \cup \{(q_8, e, q_7), (q_2, e, q_9), (q_4, e, q_9), (q_6, e, q_9)\}$$

QUESTION 3 For the automaton M

$$M = (\{q_1, q_2, q_3\},\ \{a, b\},\ s = q_1,$$

$$\Delta = \{(q_1, b, q_2), (q_1, a, q_3), (q_2, a, q_1), (q_2, b, q_1), (q_3, a, q_1), (q_3, b, q_1)\},\ F = \{q_1\})$$

1. Evaluate 4 steps, in which you must include at least one $R(i,j,0)$, in the construction of regular expression that defines $L(M)$ that uses the formulas:

$$L(M) = \bigcup \{R(1,j,n) : \ q_j \in F\}$$

$$R(i,j,k) = R(i,j,k-1) \cup R(i,k,k-1)R(k,k,k-1)^* R(k,j,k-1)$$

where n is the number of states of M, $k = 1, ...n$ and

$R(i,j,0)$ is either $\{a \in \Sigma \cup \{e\} : \ (q_i, a, q_j) \in \Delta\}$ if $i \neq j$, or is

$\{e\} \cup \{a \in \Sigma \cup \{e\} : \ (q_i, a, q_j) \in \Delta\}$ if $i = j$.

Solution

Step 1 $L(M) = R(1,1,3)$.

Step 2 $R(1,1,3) = R(1,1,2) \cup R(1,3,2)R(3,3,2)^* R(3,1,2)$.

Step 3 $R(1,1,2) = R(1,1,1) \cup R(1,2,1)R(2,2,1)^* R(2,1,1)$.

Step 4 $R(1,1,1) = R(1,1,0) \cup R(1,1,0)R(1,1,0)^* R(1,1,0)$ and

$$R(1,1,0) = \{e\} \cup \emptyset = \{e\}, \ R(1,1,1) = \{e\} \cup \{e\}\{e\}^*\{e\} = \{e\}.$$

2. Evaluate r, such that

$$\mathcal{L}(r) = L(M)$$

using the Generalized Automata Construction, as described in example 2.3.2 page 80.

Solution

Step 1: We extend M to a generalized GM, such that $L(M) = L(G(M)$ as follows:

$$GM = (\{q_1, q_2, q_3, q_4, q_5\}, \ \{a, b\}, \ s = q_4,$$

$$\Delta = \{(q_1, b, q_2), (q_1, a, q_3), (q_2, a, q_1), (q_2, b, q_1), (q_3, a, q_1), (q_3, b, q_1), (q_4, e, q_1), (q_1, e, q_5)\}, \ F = \{q_5\})$$

Step 2: Construct $GM1 \simeq GM \simeq M$ by elimination of q_2.

$$GM1 = (\{q_1, q_3, q_4, q_5\}, \ \{a, b\}, \ s = q_4,$$

$$\Delta = \{(q_1, a, q_3), (q_1, (bb \cup ba), q_1), (q_3, a, q_1), (q_3, b, q_1), (q_4, e, q_1), (q_1, e, q_5)\}, \ F = \{q_5\})$$

Step 3: Construct $GM2 \simeq GM1 \simeq GM \simeq M$ by elimination of q_3.

$$GM2 = (\{q_1, q_4, q_5\},\ \{a, b\},\ s = q_4,$$

$$\Delta = \{(q_1, (bb \cup ba), q_1), (q_1, (aa \cup ab), q_1), (q_4, e, q_1), (q_1, e, q_5)\},\ F = \{q_5\})$$

Step 4: Construct $GM3 \simeq GM2 \simeq GM1 \simeq GM \simeq M$ by elimination of q_1.

$$GM3 = (\{q_4, q_5\},\ \{a, b\},\ s = q_4,$$

$$\Delta = \{(q_4, (bb \cup ba \cup aa \cup ab)^*, q_5))\},\ F = \{q_5\})$$

Answer

$$L(M) = L(MG4) = (bb \cup ba \cup aa \cup ab)^* = ((a \cup b)(a \cup b))^*.$$

QUESTION 4 Show that the language

$$L = \{w \in \{a, b\} : w \textit{ has an equal number of } a's \textit{ and } b's\}$$

is not regular.

Solution Assume that L is regular. We know that $L_1 = a^*b^*$ is regular as it is defined by a regular expression. Hence the language $L_2 = L \cap L_1$ is regular, as the class of regular languages is closed under intersection. But obviously, $L_2 = \{a^n b^n : n \in N\}$ which was proved to be NOT regular. This contradiction proves that L is regular.

QUESTION 5 Show that the language

$$L = \{xyx^R :\ x, y \in \Sigma\}$$

is regular for any Σ.

Solution For any $x \in \Sigma, x^R = x$. Σ is finite set, hence $L = \{xyx :\ x, y \in \Sigma\}$ is also finite. Any finite set i regular.