CSE303 Q2 SAMPLE SOLUTIONS

PART 1: YES/NO QUESTIONS Circle the correct answer. Write SHORT justification.

1. The set K of states of a deterministic finite automaton can be empty.
   Justify: $s \in K$ **n**
2. Alphabet Σ of a deterministic finite automaton can be empty.
   Justify: Empty set is a finite set **y**
3. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$.
   Justify: element of $K \times \Sigma^*$ **n**
4. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ says that "$(q, w$ yields (g', w') in one step" (is a one step computation relation) iff the following condition holds
   $(q, w) \vdash_M (g', w')$ iff $\delta(q, a) = q'$.
   Justify: iff $(q, w) \vdash_M (g', w')$ iff $w = aw' \cap \delta(q, a) = q'$ **n**
5. Given $M = (K, \Sigma, \delta, s, F)$, the one step computation relation (yields in one step relation) $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a function that maps $K \times \Sigma^+$ into Σ^*.
   Justify: into $K \times \Sigma^*$ **n**
6. Given $M = (K, \Sigma, \delta, s, F)$,
   $L(M) = \{w \in \Sigma^* : \exists q \in K (s, w) \vdash^*{}_M (q, e)\}$.
   Justify: only when $q \in F$ **n**
7. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then M is also non-deterministic.
   Justify: any function is a relation and $\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K$ **y**
8. A configuration of a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$.
   Justify: definition **n**
9. A relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation relation in a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ iff the following condition holds
   $(q, w) \vdash_M (g', w')$ iff there is $u \in \Sigma \cup \{e\}$ such that $w = uw'$ and $(q, u, q') \in \Delta$.
   Justify: definition **y**
10. For any (deterministic or non-deterministic) $M = (K, \Sigma, \Delta, s, F)$, the following is true for any configurations $(q, w), (q', w')$ of M
    $(q, w) \vdash^*{}_M (q', w')\}$ iff there is a sequence

$$(q_1, w_1), (q_2, w_2), ..., (q_n, w_n),\ n \geq 1$$

of configurations, such that $q_1 = q,\ q_n = q',\ w_1 = w,\ w_n = w'$ and $(q_i, w_i) \vdash^*{}_M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$.

Justify: definitions of a reflexive, transitive closure and definition of a path **y**

11. We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a computation of the length n as a sequence

$$(q_1, w_1), (q_2, w_2), ..., (q_n, w_n),\ n \geq 1$$

of configurations, such that $q_1 = q,\ q_n = q',\ w_1 = w,\ w_n = w'$ and $(q_i, w_i) \vdash^*{}_M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$.

For any M, a computation of the length 1 exists.

Justify: path of length one always exists **y**

12. For any non-deterministic $M = (K, \Sigma, \Delta, s, F)$, there is an equivalent deterministic automata M'.

Justify: Theorem **y**

13. For any deterministic $M = (K, \Sigma, \delta, s, F)$, there is an equivalent non-deterministic automata M'.

Justify: $M = M'$ **y**

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BD: the BOOK definition of Δ in a non-deterministic $M = (K, \Sigma, \Delta, s, F)$ is:

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LD: the LECTURE definition of Δ in a non-deterministic $M = (K, \Sigma, \Delta, s, F)$ is:

1. Δ is finite

2. $\Delta \subseteq K \times \Sigma^* \times K$.

OBSERVE that we have to say in this case that Δ is finite because Σ^* is an infinite set.

SOLVING PROBLEMS you can use any of these definitions.

PART 2: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton M such that

$$L(M) = \{w \in \{a, b\}^* :\ w\ \textit{does not contain three consecutive}\ b's\}.$$

Draw a state diagram and specify all components K, Σ, δ, s, F of M. Justify your construction.

Components of M are:

$$K = \{q_0, q_1, q_2, q_3\},\ \Sigma = \{a, b\},\ s = q_0,\ F = \{q_0, q_1, q_2\}$$

$$\delta = \{(q_0, a, q_0), (q_0, b, q_1), (q_1, a, q_0), (q_1, b, q_2), (q_2, a, q_0),$$
$$(q_2, b, q_3), (q_3, a, q_3), (q_3, b, q_3)\}$$

q_3 is a TRAP state.

Some elements of $L(M)$ are: $e, a, aa, aaa, b, bb, ba, baa, abba, aabba, ...$

QUESTION 2

1. Construct a non-deterministic finite automaton M, such that

$$L(M) = (aba \cup ba)^*.$$

You can use **BD**definition or **LD** definition. Draw a state diagram and specify all components K, Σ, Δ, s, F. Justify your construction by listing strings accepted the state diagram of M.

Solution We use **LD** lecture definition.

Components of M are:

$$K = \{q_0\},\ \Sigma = \{a, b\},\ s = q_0,\ F = \{q_0\}$$

$$\Delta = \{(q_0, aba, q_0), (q0, ba, q_0)\}$$

Some elements of $L(M)$ are: $aba, abaaba, ba, baba, ababa, baaba, ...$

2. Construct a deterministic finite automaton M, such that

$$L(M) = (aba \cup ba)^*.$$

Solution

Components of M are:

$$K = \{q_0, q_1, q_2, q_3, q_4\},\ \Sigma = \{a, b\},\ s = q_0,\ F = \{q_0\}$$

$$\delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_4), (q_1, b, q_2), (q_2, a, q_0), (q_2, b, q_4),$$
$$(q_3, a, q_0), (q_3, b, q_4), (q_4, a, q_4), (q_4, b, q_4), \}$$

q_4 is a TRAP state.

Some elements of $L(M)$ are: $aba, ba, ababa, baba, baaba, \ldots$.

QUESTION 3 Given $M = (\{q_0, q_1, q_2\},\ \{a, b\},\ q_0,\ \Delta,\ \{q_2\}\ )$ for

$$\Delta = \{(q_0, bab, q_0), (q_0, e, q_1), (q_0, b, q_1), (q_1, ba, q_2)\}$$

Transform M into and equivalent $M_1 = (K_1,\ \{a, b\},\ q_0,\ \Delta_1, \{q_2\})$ with $\Delta_1 \subseteq \Sigma \cup \{e\}$.

Solution

Components of M_1 are:

$$K_1 = \{q_0, q_1, q_2, p_1, p_2, p_3\},\ \Sigma = \{a, b\},\ s = q_0,\ F_1 = \{q_2\}$$

$$\Delta_1 = \{(q_0, b, p_1), (p_1, a, p_2), (p_2, b, q_0), (q_0, e, q_1), (q_0, b, q_1), (q_1, b, p_3), (p_3, a, q_2)\}$$

QUESTION 4 Let M be defined as follows

$$M = (\{q_0, q_1, q_2\}, \{b\},\ s = q_0,\ \Delta,\ F = \{q_2\}\ )$$

and

$$\Delta = \{(q_0, b, q_0), (q_0, e, q_1), (q_0, b, q_1), (q_1, b, q_2)\}$$

Describe $L(M)$

Solution

$$L(M) = b^+$$

Write all steps of the general method of transformation a NDFA M, into an equivalent M', which is a DFA. Compute only states reachable from $S = E(s)$, i.e. only the essential part of M'.

Step 1: Evaluate $\delta(E(q_0), b)$.

Step $i+1$: Evaluate δ on all states that result from step i.

Reminder: $E(q) = \{p \in K : (q, e) \overset{*,M}{\longmapsto} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Solution

Step 1:

$$E(q_0) = \{q_0, q_1\},\ E(q_1) = \{q_1\},\ E(q_2) = \{q_2\}$$

$$\delta(\{q_0, q_1\}, b) = E(q_0) \cup E(q_2) = \{q_0, q_1, q_2\}$$

Step 2:

$$\delta(\{q_0, q_1, q_2\}, b) = E(q_0) \cup E(q_2 1) \cup \emptyset = \{q_0, q_1, q_2\}$$

all other states $Q \in 2^{\{q_0, q_1, q_2\}}$ are TRAP states.

$$L(M') = b^+$$

QUESTION 5 For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_0, q_2\}$ and

$$\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}$$

Write 3 steps of the general method of transformation a NDFA M, into an equivalent M', which is a DFA, where M is given by a following state diagram.

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step i+1: Evaluate δ on all states that result from step i.

Reminder: $E(q) = \{p \in K : (q, e) \vdash^*{}_M (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup \{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Solution

Step 1:

$$E(q_0) = \{q_0\},\ E(q_1) = \{q_1\},\ E(q_2) = \{q_2\}$$

$$\delta(\{q_0\}, a) = E(q_1) = \{q_1\} \quad \delta(\{q_0\}, b) = \emptyset$$

Step 2:

$$\delta(\emptyset, a) = \emptyset,\ \delta(\emptyset, a) = \emptyset, \delta(\{q_1\}, a) = \emptyset,\ \delta(\{q_1\}, b) = E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$

Step 3:

$$\delta(\{q_0, q_2\}, a) = E(q_1) \cup E(q_0) = \{q_0, q_1\}, \quad \delta(\{q_0, q_2\}, b) = \emptyset$$

Step 4:

$$\delta(\{q_0, q_1\}, a) = \emptyset \cup E(q_1) = \{q_1\},\ \ \delta(\{q_0, q_1\}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$