CSE303 Q2 SOLUTIONS

PART 1: YES/NO QUESTIONS Circle the correct answer. Write SHORT justification. Answers without justification will not receive credit.

1. The set F of final states of any deterministic finite automaton is always non-empty.

   Justify: the definition says that F is a finite set, i.e. can be empty, hence for some M. $L(M) = \emptyset$.

   n

2. Alphabet Σ of any deterministic, or non-deterministic finite automaton is always non-empty.

   Justify: the definition says that Σ is an alphabet, what means that is a finite set, i.e. can be empty, hence for some M. $L(M) = \emptyset$.

   n

3. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$.

   Justify: a configuration is an element of $K \times \Sigma^*$.

   n

4. Given a non-deterministic automaton $M = (K, \Sigma, \Delta, s, F)$ as defined in the book, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds

   $(q, aw) \vdash_M (q', w)$ iff $(q, a, q') \in \Delta$ and $a \in \Sigma \cup \{e\}$.

   Justify: it is the book definition

   y

5. For any $M = (K, \Sigma, \delta, s, F)$, $L(M) \neq \emptyset$
   Justify: for any M, such that $F = \emptyset$ or $\Sigma = \emptyset$ we have that $L(M) = \emptyset$.

   n

6. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.
   Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$.

   y

7. A configuration of a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$.
   Justify: by definition

   y

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

PART 2: Few Very Short Questions For the automata M below

1. State and explain whether M represents a deterministic or a non-deterministic automaton.

2. Write down a regular expression representing $L(M)$.

Q1: M1 has components: $K = \{q\}$, $s = q, \Sigma = \emptyset, \delta = \emptyset, F = \emptyset$.

Solution

1. $M1$ is deterministic.

2. $L(M1) = \emptyset$.

Q2: M2 has components: $K = \{q_0, q_1, q_2\}$, $s = q_0, \Sigma = \{a, b\}, \delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$, $F = \{q_1\}$.

Solution:

1. $M2$ is non-deterministic; δ is not a function with the domain $K \times \Sigma$. It can be completed to a function by adding some trap states. But the trap states information was not it was not stated in the problem.

2. $L(M2) = aa^*$.

Q3: M3 $K = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$, $F = \{q_1, q_2\}$.

Solution $M3$ is NOT an automaton. It does not have the INITIAL state!

PART 3: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton M such that

$$L(M) = \{w \in \{a,b\}^* : \textit{ w always contains the substring aab or bba}\}.$$

Specify all components K, Σ, δ, s, F of M. Justify your construction.

Solution

Components of M are:

$$K = \{q_0, q_1, q_2, q_3, q_4, q_5\},\ \Sigma = \{a,b\},\ s = q_0,$$

$$\delta = \{(q_0, b, q_1), (q_0, a, q_3), (q_1, a, q_3), (q_1, b, q_2), (q_2, a, q_5), (q_2, b, q_2),$$
$$(q_3, a, q_4), (q_3, b, q_1), (q_4, a, q_4), (q_4, b, q_5), (q_5, a, q_5), (q_5, b, q_5)\},$$
$$F = \{q_5\}.$$

Some elements of $L(M)$ are:

$$bbaba,\ abaab,\ ababaab,\ aaaabaaba,\ bbbbbaa,$$

QUESTION 2 Construct a non-deterministic finite automaton M, such that

$$L(M) = (ba \cup b)^* \cup (bb \cup a)^*.$$

Solution

Components of M are:

$$K = \{q_0, q_1, q_2\},\ s = q_0,\ \Sigma = \{a,b\},$$

$$\Delta = \{(q_0, e, q_1), (q_0, e, q_2), (q_1, ba, q_1), (q_1, b, q_1), (q_2, bb, q_2), (q_2, a, q_2)\},$$
$$F = \{q_1, q_2\}.$$

Some elements of $L(M)$ are:

$$bab, bba, babb, bbaa, bbaabb, babbba, ...$$

QUESTION 3 Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3, \}$, $s = q_0$
$\Sigma = \{a, b, c\}$, $F = \{q_0, q_2, q_3\}$ and
$\Delta = \{(q_0, ab, q_0), (q_0, c, q_1), (q_1, bc, q_2), (q_0, b, q_2), (q_2, a, q_2), (q_2, e, q_3), (q_3, b, q_3)\}$.

1. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps.

Solution

$$L = (ab)^* \cup (ab)^*cbc \cup (ab)^*cbca^* \cup (ab)^*cbca^*b^* \cup (ab)^*b \cup (ab)^*ba^*b^*.$$

Observe that $e \in L$ as $q_0 \in F$, so we must have $(ab)^*$ alone in the L.

We SIMPLIFY L as follows.

$$L = (ab)^* \cup (ab)^*cbca^*b^* \cup (ab)^*ba^*b^* = (ab)^*(e \cup cbc \cup b)a^*b^*.$$

We used the property:

$$LL_1 \cup LL2 = L(L_1 \cup L_2).$$

2. Write down (you can draw the diagram) an automata M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution We apply the "stretching" technique to M and the new M' is is as follows.

$$M' = (K \cup \{p_1, p_2\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$$

for $K = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b\},\ F = \{q_0, q_2, q_3\}$ and
$\Delta' = \{(q_0, c, q_1), (q_0, b, q_2), (q_2, a, q_2), (q_2, e, q_3), (q_3, b, q_3)\} \cup \{(q_0, a, p_1), (p_1, b, q_0), (q_1, b, p_2), (p_2, c, q_2)\}.$

Problem 4 For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_0, q_2\}$ and

$$\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}$$

Write 3 steps of the general method of transformation a NDFA M, into an equivalent M', which is a DFA, where M is given by a following state diagram.

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step i+1: Evaluate δ on all states that result from step i.

Reminder: $E(q) = \{p \in K : (q, e) \vdash^*{}_M (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup \{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Solution

Step 1:

$$E(q_0) = \{q_0\},\ E(q_1) = \{q_1\},\ E(q_2) = \{q_2\}$$

$$\delta(\{q_0\}, a) = E(q_1) = \{q_1\} \quad \delta(\{q_0\}, b) = \emptyset$$

Step 2:

$$\delta(\emptyset, a) = \emptyset,\ \delta(\emptyset, a) = \emptyset, \delta(\{q_1\}, a) = \emptyset,\ \delta(\{q_1\}, b) = E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$

Step 3:

$$\delta(\{q_0, q_2\}, a) = E(q_1) \cup E(q_0) = \{q_0, q_1\}, \quad \delta(\{q_0, q_2\}, b) = \emptyset$$

Step 4:

$$\delta(\{q_0, q_1\}, a) = \emptyset \cup E(q_1) = \{q_1\},\ \delta(\{q_0, q_1\}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$