CSE303 Q2 PRACTICE SOLUTIONS

PART 1: YES/NO QUESTIONS Circle the correct answer. Write SHORT justification.

1. The set K of states of any deterministic finite automaton is always non-empty
   Justify: $s \in K$

   y

2. Alphabet Σ of any deterministic finite automaton is always non-empty

   Justify: An alphabet Σ is any FINITE set, hence it can be empty.

   n

3. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^*$.

   Justify: this is definition

   y

4. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds

   $(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$.

   Justify: Proper condition is:
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$.

   n

5. Given $M = (K, \Sigma, \delta, s, F)$ we define
   $L(M) = \{w \in \Sigma^* : \ \exists q \in K((s, w) \vdash^*{}_M (q, e))\}$.
   Justify: Must be: $\exists q \in F((s, w) \vdash^*{}_M (q, e))$.

   n

6. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then M is also non-deterministic.
   Justify: The function δ is a (special) relation on $K \times \Sigma \times K$, i.e. $\delta = \Delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K \subseteq K \times \Sigma^* \times K$.

   y

7. A configuration of a non- deterministic finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^*$.
   Justify: by definition

   y

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K.$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

A VERY SHORT QUESTION Given the automaton M with the following components:

$\Sigma = \{a, b, c\}$, $K = \{q_0, q_1, q_2\}$, $s = q_0$, $F = \{q_2\}$.
We define Δ as follows.
$\Delta = \{(q_0, abc, q_1), (q_1, e, q_2)), (q_0, a, q_2)\}$

1. State and explain whether M represents a deterministic or a non-deterministic automaton.

Solution M is non-deterministic. Δ is not a function on $K \times \Sigma$,, also $\Delta \subseteq K \times \Sigma^* \times K$ (Lecture definition).

2. Write down a regular expression representing $L(M)$.

Solution

$$L(M) = abc \cup a$$

PART 2: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton M such that

$$L(M) = \{w \in \{a, b\}^* : \textit{ neither } bb \textit{ nor } aa \textit{ is a substring of } w\}.$$

Draw a state diagram and specify all components K, Σ, δ, s, F of M. Justify your construction.

Solution

Components of $M = (K, \Sigma, \delta, s, F)$ are:
$\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, q_3 is a trap state, $F = \{q_0, q_1, q_2\}$.
We define δ on non-trap states as follows.
$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$,
$\delta(q_1, b) = q_2$,
$\delta(q_2, a) = q_1$.

M **accepts** strings $a, aba, ababa....$ or $b, bab, baba..$ etc and never aa, bb, etc...

QUESTION 2 For the automata M defined below describe the property defining $L(M)$.

Components of M are:
$\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_1\}$.
We define δ as follows.

$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$,
$\delta(q_1, a) = q_0$, $\delta(q_1, b) = q_3$,
$\delta(q_2, a) = q_3$, $\delta(q_2, b) = q_0$,
$\delta(q_3, a) = q_2$, $\delta(q_3, b) = q_1$.

Solution

Language of M is:

$$L(M) = \{w \in \Sigma^* : \ w\ has\ an\ odd\ number\ of\ a's\ and\ an\ even\ number\ of\ b's\}.$$

QUESTION 3 Use book or lecture definition (specify which are you using) to construct a non-deterministic finite automaton M, such that

$$L(M) = (ab)^*(ba)^*.$$

Draw a state diagram and specify all components K, Σ, Δ, s, F of M. Justify your construction by listing some strings accepted by the state diagram.

Solution 1 We use the lecture definition.

Components of M are: $\Sigma = \{a, b\}$, $K = \{q_0, q_1\}$, $s = q_0$, $F = \{q_0, q_1\}$.
We define Δ as follows.
$\Delta = \{(q_0, ab, q_0), (q_0, e, q_1), (q_1, ba, q_1)\}$.

Strings accepted : $ab, abab, abba, ababba, ababbaba, ...$

Solution 2 We use the book definition.

Components of M are: $\Sigma = \{a, b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $F = \{q_2\}$.
We define Δ as follows.
$\Delta = \{(q_0, a, q_1), (q_1, b, q_0), (q_0, e, q_2), (q_2, b, q_3), (q_3, a, q_2)\}$.

Strings accepted : $ab, abab, abba, ababba, ababbaba, ...$

QUESTION 4 Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3, \}$, $s = q_0$
$\Sigma = \{a, b, c\}$, $F = \{q_3\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$.

1. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps.

Solution $L(M) = (abc)^*abbb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa (abc)^*(abbb \cup baa)$.

We used the property:

$$LL_1 \cup LL2 = L(L_1 \cup L_2).$$

2. Write down (you can draw the diagram) an automata M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution

Solution We apply the "stretching" technique to M and the new M' is is as follows.

$$M' = (K \cup \{p_1, p_2, ..., p_5\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_3\}$ and
$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1), (q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$.

QUESTION 5 Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_0, q_2\}$ and

$$\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_1, b, q_0), (q_2, a, q_0)\}.$$

Write 4 steps of the general method of transformation a NDFA M, into an equivalent M', which is a DFA. Reminder: $E(q) = \{p \in K : (q, e) \vdash^*{}_M (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup \{E(p) : \quad \exists_{q \in Q} (q, \sigma, p) \in \Delta \ , \ p \in K\}$$

Step 1: Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

Step i+1: Evaluate δ on all states that result from step i.

Solution

Step 1:

$$E(q_0) = \{q_0\}, \ E(q_1) = \{q_1\}, \ E(q_2) = \{q_2\}$$

$$\delta(\{q_0\}, a) = E(q_1) = \{q_1\} \quad \delta(\{q_0\}, b) = \emptyset$$

Step 2:

$$\delta(\emptyset, a) = \emptyset, \ \delta(\emptyset, a) = \emptyset, \delta(\{q_1\}, a) = \emptyset, \ \delta(\{q_1\}, b) = E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$

Step 3:

$$\delta(\{q_0, q_2\}, a) = E(q_1) \cup E(q_0) = \{q_0, q_1\}, \quad \delta(\{q_0, q_2\}, b) = \emptyset$$

Step 4:

$$\delta(\{q_0, q_1\}, a) = \emptyset \cup E(q_1) = \{q_1\}, \ \delta(\{q_0, q_1\}, b) = \emptyset \cup E(q_0) \cup E(q_2) = \{q_0, q_2\} \in F'$$