NAME

ID:

MY POINTS ARE:

TAKE test as a practice - and correct it yourself to see how much points you would get.

Solutions to all problems and questions are somewhere on our webpage! so you CAN correct yourself- but do it ONLY AFTER you complete it all by yourself.

This is the goal of the PRACTICE TEST!

PLEASE SUBMIT your SOLUTIONS that have been CORRECTED BY YOU - Write corrections in RED. You WILL GET 15 points for THAT! even if all problems you solved were first wrong- and then CORRECTED!

Write a sum of POINTS you give yourself for your solutions -after you check your answers for corrections.

The real midterm will have less problems; I will make sure you will be able to complete it within 1 hour and 15 minutes.

BRING YOUR solved-corrected TEST to class on Monday, March 19

I WILL POST THE SOLUTIONS on MONDAY for you to STUDY for MIDTERM

MIDTERM is March 21

I will NOT ACCEPT SOLUTIONS after MONDAY, March 19

1 YES/NO questions

Circle the correct answer (each question is worth 1pt) Write SHORT justification.

1. For any function $f$ from $A \neq \emptyset$ onto $A$, $f$ has property $f(a) \neq a$ for certain $a \in A$.
   
   Justify:

   y  n

2. For any binary relation $R \subseteq A \times A$, $R^{-1}$ exists.
   
   Justify:

   y  n

3. All infinite sets have the same cardinality.
   
   Justify:

   y  n

4. Set $A$ is uncountable iff $R \subseteq A$ ($R$ is the set of real numbers).
   
   Justify:

   y  n
5. Let $A \neq \emptyset$ such that there are exactly 25 partitions of $A$. It is possible to define 20 equivalence relations on $A$.

   Justify: \hspace{100pt} y n

6. There is a relation that is equivalence and order at the same time.

   Justify: \hspace{100pt} y n

7. Let $A = \{n \in N : n^2 + 1 \leq 15\}$. It is possible to define 8 alphabets $\Sigma \subseteq A$.

   Justify: \hspace{100pt} y n

8. There is exactly as many languages over alphabet $\Sigma = \{a\}$ as real numbers.

   Justify: \hspace{100pt} y n

9. Let $\Sigma = \{a, b\}$. There are more than 20 words of length 4 over $\Sigma$.

   Justify: \hspace{100pt} y n

10. $L^* = \{w_1 \ldots w_n : w_i \in L, i = 1, 2, \ldots, n, n \geq 1\}$.

    Justify: \hspace{100pt} y n

11. $L^+ = LL^*$.

    Justify: \hspace{100pt} y n

12. $L^+ = L^* \setminus \{e\}$.

    Justify: \hspace{100pt} y n

13. If $L = \{w \in \{0, 1\}^* : w$ has an unequal number of 0’s and 1’s $\}$, then $L^* = \{0, 1\}^*$.

    Justify: \hspace{100pt} y n

14. For any languages $L_1, L_2$,

    \[ L_1^* = L_2^* \text{ if and only if } L_1 = L_2 \]

    Justify: \hspace{100pt} y n

15. For any languages $L_1, L_2$, $(L_1 \cup L_2) \cap L_1 = L_1$.

    Justify: \hspace{100pt} y n

16. For any languages $L_1, L_2$, $(L_1 \cup L_2)^* = L_1^*$.

    Justify: \hspace{100pt} y n

17. $((\emptyset^* \cap a) \cup b^*) \cap \emptyset^*$ describes a language with only one element.

    Justify: \hspace{100pt} y n
18. \((\emptyset \cap a^*) \cup a^*\) is a finite regular language.
   Justify: 
   y n

19. \((\{a\} \cap \{e\}) \cap \{ab\}^*\) is a finite regular language.
   Justify: 
   y n

20. Any regular language has a finite description.
   Justify: 
   y n

21. Any finite language is regular.
   Justify: 
   y n

22. Every deterministic automaton is also non-deterministic.
   Justify: 
   y n

23. The set of all configurations of a given finite state automaton is always non-empty.
   Justify: 
   y n

24. Let \(M\) be a finite state automaton, \(L(M) = \{\omega \in \Sigma^* : (q, \omega) \xrightarrow{\epsilon, M} (s, e)\}\).
   Justify: 
   y n

25. For any automaton \(M\), \(L(M) \neq \emptyset\).
   Justify: 
   y n

26. \(L(M_1) = L(M_2)\) iff \(M_1, M_2\) are deterministic.
   Justify: 
   y n

27. DFA and NDFA compute the same class of languages.
   Justify: 
   y n

28. Let \(M_1\) be a deterministic, \(M_2\) be a nondeterministic FA, \(L_1 = L(M_1)\) and \(L_2 = L(M_2)\) then there is a deterministic automaton \(M\) such that \(L(M) = (L^* \cup (L_1 - L_2)^*)L_1\)
   Justify: 
   y n

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: \(M = (K, \Sigma, \Delta, s, F)\) is non-deterministic when \(\Delta \subseteq K \times (\Sigma \cup \{\epsilon\}) \times K\).
   OBSERVE that \(\Delta\) is always finite because \(K, \Sigma\) are finite sets.

LECTURE DEFINITION: \(M = (K, \Sigma, \Delta, s, F)\) is non-deterministic when \(\Delta\) is finite and \(\Delta \subseteq K \times \Sigma^* \times K\).
   OBSERVE that we have to say in this case that \(\Delta\) is finite because \(\Sigma^*\) is infinite.

SOLVING PROBLEMS you can use any of these definitions.
2 Problems

PROBLEM 1 (10 pts)

Let \( L \) be a language defines as follows

\[
L = \{ w \in \{a, b\}^* : \text{every } a \text{ is either immediately preceded or followed by } b \}\.
\]

1. Describe a regular expression \( r \) such that \( L(r) = L \). Explain shortly your solution.

2. Construct a finite state automata \( M \), such that \( L(M) = L \).

State Diagram of \( M \) is:

Some elements of \( L(M) \) as defined by the state diagram are:

Components of \( M \) are:
**PROBLEM 2** (6 pts)

1. Let $M$ be a deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

2. Let $M$ be a non-deterministic finite automaton. Under exactly what conditions $e \in L(M)$?

**PROBLEM 3** (16 pts) Let

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, $\Sigma = \{a, b\}$, $F = \{q_1, q_2, q_3\}$, and

$$\Delta = \{(q_0, a, q_1), (q_0, b, q_3), (q_1, a, q_2), (q_1, b, q_1), (q_3, a, q_3), (q_3, b, q_2)\}$

1. Draw the State Diagram of $M$.

2. List few elements of $L(M)$.

3. Write a regular expression for the language accepted by $M$. Explain and simplify the solution.
4. Define a deterministic $M'$ such that $M \approx M'$, i.e. $L(M) = L(M')$. 

State Diagram of $M'$ is:

Justify why $M \approx M'$.

PROBLEM 4 (15 pts) 
Let $M$ be defined as follows 

$M = (K, \Sigma, s, \Delta, F)$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$ 
$
\Sigma = \{a, b, c\}$, $F = \{q_0, q_2, q_3\}$ and 
$
\Delta = \{(q_0, abc, q_0), (q_0, a, q_1), (q_0, e, q_3), (q_1, bc, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3), (q_3, a, q_3)\}$.

1. Draw the State Diagram of $M$.

2. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps. Does $e \in L(M)$?
3. Write down (you can draw the diagram) an automata $M'$ such that $M' \equiv M$ and $M'$ is defined by the BOOK definition.

**PROBLEM 5** (15 pts.) For $M$ defined as follows

$$M = (K, \Sigma, s, \Delta, F)$$

for $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$
$
\Sigma = \{a, b\}$, $F = \{q_2\}$ and

$$\Delta = \{(q_0, a, q_3), (q_0, e, q_3), (q_0, b, q_1), (q_0, e, q_1), (q_1, a, q_2), (q_2, b, q_3), (q_2, e, q_3)\}$$

1. Draw the State Diagram of $M$.

2. Write 2 steps of the general method of transformation a NDFA $M$, into an equivalent $M'$, which is a DFA, where $M$ is given by a following state diagram.

**Step 1:** Evaluate $\delta(E(q_0), a)$ and $\delta(E(q_0), b)$.

**Step 2:** Evaluate $\delta$ on all states that result from step 1.

**Reminder:** $E(q) = \{p \in K : (q, e) \xrightarrow{M} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q, (q, \sigma, p) \in \Delta\}.$$