CSE303 Q2 SOLUTIONS Spring 2018

PART 1: YES/NO QUESTIONS

1. The set F of final states of any deterministic finite automaton is always non-empty.
   Justify: the definition says that F is a finite set, i.e. can be empty, hence for some M, $L(M) = \emptyset$. **n**
2. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$
   Justify: it is any element of $K \times \Sigma^*$ **n**
3. Given an automaton $M = (K, \Sigma, \delta, s, F)$, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a **transition relation** iff the following condition holds
   $(q, aw) \vdash_M (q', w)$ iff $\delta(q', a) = q$.
   Justify: Proper condition is: $(q, aw) \vdash_M (q', w)$ iff $\delta(q, a) = q'$ **n**
4. For any $M = (K, \Sigma, \delta, s, F)$, $L(M) \neq \emptyset$
   Justify: for any M, such that $F = \emptyset$ we have that $L(M) = \emptyset$
   n
5. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.
   Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$
   y

PART 2: Few Very Short Questions

Q1: M1 has components: $K = \{q\}$, $s = q, \Sigma = \emptyset, \delta = \emptyset, F = \emptyset$.

Solution

1. $M1$ is deterministic

2. $L(M1) = \emptyset$

Q2: M2 has components: $K = \{q_0, q_1, q_2\}$, $s = q_0, \Sigma = \{a, b\}$, $F = \{q_1\}$
$\delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$

Solution

1. $M2$ is non-deterministic; δ is not a function with the domain $K \times \Sigma$. It can be completed to a function by adding some trap states. But the trap states information was not stated in the problem - so **M2** is NDFA

2. $L(M2) = aa^*$

Q3 M3 has components: $K = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}$, $F = \{q_1, q_2\}$
$\Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$

Solution

M3 is NOT an automaton. It does not have the INITIAL state!

PART 3: PROBLEMS

QUESTION 1 The components of M represent a deterministic finite automaton M with some trap states.

Components of M are:
$K = \{q_0, q_1, q_2, q_3\},\ s = q_0,\ \Sigma = \{a, b\},\ F = \{q_1, q_3\}$
$\delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_3)\}$

Complete the components to a full definition of a deterministic M by adding trap state(s)

Solution We add a new state q_4 and extend δ to function δ_1 such that
$\delta_1 : (K \cup \{q_4\}) \times \Sigma \longrightarrow K \cup \{q_4\}$ as follows

$$\delta_1 = \delta \cup \{(q_0, b, q_4), (q_1, a, q_14), (q_3, a, q_4), (q_3, b, q_4)\}$$

Write a regular expression or a property defining $L(M)$.

Solution $L(M) = a \cup aba^*b$

QUESTION 2 Construct a non-deterministic finite automaton M, such that

$$L(M) = (ba \cup b)^* \cup (bb \cup a)^*.$$

Solution Some elements of $L(M)$ are: $bab, bba, babb, bbaa, bbaabb, babbba, ...$

Components of M are:

$$K = \{q_0, q_1, q_2\},\ s = q_0,\ \Sigma = \{a, b\},$$

$$\Delta = \{(q_0, e, q_1), (q_0, e, q_2), (q_1, ba, q_1), (q_1, b, q_1), (q_2, bb, q_2), (q_2, a, q_2)\},$$

$$F = \{q_1, q_2\}$$

QUESTION 3

Let $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0, q_1, q_2\},\ s = q_0,\ \ \Sigma = \{a, b, c\},\ F = \{q_1, q_2\}$ and $\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_0, b, q_2)\}$

Draw the diagram an automaton M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution

We apply the **"stretching"** technique to M and the new M' is is as follows.

$M' = (K \cup \{p_1, p_2, p_3\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1),\ (p_1, b, p_2),\ (p_2, c, q_0),\ (q_0, b, q_2),\ (q_0, a, p_3),\ (p_3, b, q_1)\}$