CSE303 Q1 Solutions Spring 2018

PART 1: YES/NO QUESTIONS

Circle the correct answer. **Write** SHORT justification.

1. $\{1, \{\emptyset\}, 2\} \cap 2^{\emptyset} \neq \emptyset$
   Justify: $2^{\emptyset} = \{\emptyset\}$ and $\emptyset \notin \{1, \{\emptyset\}, 2\}$ **n**
2. The relation $R = \{(n, m) : \ n \in N$ and $m \in \{0, 3\}\} isafunction$
   Justify: R is not a function. For example $(0,0) \in R$, $(0,3) \in R$, $(1,0) \in R$, $(1,3) \in R$. In fact for any $n \in N$, $(n, 0) \in R$ and $(n, 3) \in R$. **n**
3. If the A is uncountable, then $|A| = \mathcal{C}$
   Justify: The set 2^R of all subsets of real numbers R is uncountable, but by Cantor Theorem we have that $|2^R| > |R| = \mathcal{C}$. **n**
4. The set $A = \{x \in N : \ x < 0\}$ is infinite
   Justify: A is finite as $\{x \in N : \ x < 0\} = \emptyset$ **n**
5. The set $A = \{\emptyset, \{\emptyset\}\}$ is an alphabet
   Justify: A is a finite set and any finite set is an alphabet by definition **y**
6. Let $\Sigma = \{\emptyset\}$. There are uncountably many languages over Σ.
   Justify: $\Sigma \neq \emptyset$ as $\emptyset$ is element of Σ. Hence there are infinitely countably many elements in $\Sigma*$ and uncountably many subsets of Σ^*.
   In fact exactly as many as real numbers as $|2^{\Sigma^*}| = |R| = \mathcal{C}$. **y**
7. For any languages $L_1, \ L_2, \ L$ over $\Sigma \neq \emptyset$
   $(L_1 \cup L_2) \circ L = (L_1 \circ L) \cup (L_2 \circ L)$
   Justify: We proved that concatenation is distributive over union of languages **y**
8. $L^* = \{w_1...w_n : w_i \in L, i = 1, 2, ..n, n \geq 1\}$
   Justify: This is definition of L^+; Kleene Star must have condition $n \geq 0$. **n**
9. Regular language is a regular expression.
   Justify: Regular language is *defined* by a regular expression. **n**
10. For any language L over an alphabet Σ, $L^+ = L \cup L^*$.
    Justify: only when $e \in L$ as $e \in L^*$ always. **n**

PART 2

QUESTION 1

Given an alphabet $\Sigma = \{a, b\}$ and a regular expression $\alpha = ab^\star \cup a^\star b \cup (a \cup b)^\star$.

1.Evaluate $L = \mathcal{L}(\alpha)$.
2. Give a property describing the language L determined by α,

Solution

1. We evaluate

$$L = \mathcal{L}(ab^\star \cup a^\star b \cup (a \cup b)^\star) = \mathcal{L}(ab^\star) \cup \mathcal{L}(a^\star b) \cup (\mathcal{L}(a) \cup \mathcal{L}(b))^\star = \{a\}\{b\}^* \cup \{b\}\{a\}^* \cup \{a, b\}^*$$

2. Observe that $\{a\}\{b\}^* \cup \{b\}\{a\}^* \subseteq \{a, b\}^*$ hence the language is

$$L = \{w : w \in \{a, b\}^\star\} = \Sigma^\star$$

QUESTION 2

Let $\Sigma = \{a, b\}$. Let $L \subseteq \Sigma^\star$ be defined as $L = \{w \in \Sigma^\star :$ w contains an EVEN the number of b 's$\}$

Write a regular expression α, such that $\mathcal{L}(\alpha) = L$. You can use shorthand notation. Explain shortly your answer.

Solution

$$\alpha = a^\star(a^\star ba^\star ba^\star)^\star$$

Justification

The regular expression $a^\star ba^\star ba^\star$ describes a string $w \in \Sigma^\star$ with **exactly two** b 's.

The regular expression

$$(a^\star ba^\star ba^\star)^\star$$

represents multiples of $w \in \Sigma^\star$ with **exactly two** b 's and hence represents words in which a number of b 's is an even number.

0 is an even number, so we need to add the case of zero number of b 's, i.e. we need to include words $e, a, aa, aaa, ...$

We do so by concatenating $(a^\star ba^\star ba^\star)^\star$ with a^* and get

$$L = a^\star(a^\star ba^\star ba^*)^\star \quad \text{or} \quad L = (a^\star ba^\star ba^\star)^\star a^\star$$