CSE303 MIDTERM SOLUTIONS

1 YES/NO questions

1. For any function f from $A \neq \emptyset$ onto A, f has property $f(a) \neq a$ for certain $a \in A$.
   Justify: $f(x) = x$ is always "onto". **n**

2. All infinite sets have the same cardinality.
   Justify: $|N| \neq |R|$ and N (natural numbers) and R (real numbers) are infinite sets. **n**

3. $\{\{a,b\}\} \in 2^{\{a,b,\{a,b\}\}}$
   Justify: $\{\{a,b\}\} \subseteq \{a,b,\{a,b\}\}$. **y**

4. For any binary relation $R \subseteq A \times A$, R^{-1} exists.
   Justify: The set $R^{-1} = \{(b,a) : \ (a,b) \in R\}$ always exists. **y**

5. Regular language is a regular expression.
   Justify: Regular language is a language defined by a regular expression. **n**

6. $L^+ = \{w_1...w_n : w_i \in L, i = 1,2,..n, n \geq 1\}$.
   Justify: definition **y**

7. $L^+ = L^* - \{e\}$.
   Justify: only when $e \notin L$. When $e \in L$ we get that $e \in L^+$ and $e \notin L^* - \{e\}$. **n**

8. For any languages L_1, L_2, $(L_1 \cap L_2) \cup L_2 = L_2$.
   Justify: $L_1 \cap L_2 \subseteq L_2$ and languages are sets. **y**

9. $(\emptyset^* \cap b^*) \cup \emptyset^*$ describes a language with only one element.
   Justify: $(\{e\} \cap \{b\}^*) \cup \{e\} = \emptyset \cup \{e\} = \{e\}$ **y**

10. For any M,$L(M) \neq \emptyset$ iff the set F of its final states is non-empty.
    Justify: Let M be such that $\Sigma = \emptyset, F \neq \emptyset, s \notin F$, we get $L(M) = \emptyset$. **n**

11. A configuration of any finite automaton $M = (K, \Sigma, \Delta, s, F)$ is any element of $K \times \Sigma^* \times K$.
    Justify: it is element of $K \times \Sigma^*$ (lecture definition) **n**

12. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.
    Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$ **y**

13. Let M be a finite state automaton, $L(M) = \{\omega \in \Sigma^* : (s,\omega) \overset{*,M}{\longmapsto} (q,e)\}$.
    Justify: only when $q \in F$ **n**

14. $L(M_1) = L(M_2)$ iff M_1, M_2 are finite automata.
    Justify: one can have 2 automata that accept different languages. **n**

15. DFA and NDFA recognize the same class of languages.
    Justify: theorem proved in class **y**

2 Two definitions of a non-deterministic automaton

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

3 Very short questions (25pts)

For all state diagrams below do the following.

1. Determine whether it defines a finite state automaton.

2. Determine whether it is a deterministic / non-deterministic automaton.

3. Write full definition of M by listing all its components.

4. Describe the language by writing a regular expression or a property that defines it.

Q1 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0\} = F,\ s = q_0,\ \Sigma = \emptyset, \Delta = \emptyset$. M is deterministic and

$$L(M) = \{e\} \neq \emptyset$$

Q2 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1\},\ s = q_0, F = \{q_0\}, \Delta = \{(q_0, a, q_1), (q_1, b, q_0)\}$. M is non deterministic; Δ is not a function on $K \times \Sigma$.

$$L(M) = (ab)^*$$

Q3 Solution: $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1, q_2\}, F = \{q_1\}$,
$\Delta = \{(q_0, a, q_1), (q_0, b, q_1), (q_1, a, q_1), (q_1, e, q_2), (q_2, ab, q_2)\}$. It is NOT an automaton. It has no initial state.

Q4 $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1, q_2, q_3\},\ s = q_0, F = \emptyset$,
$\Delta = \{(q_0, a, q_1), (q_1, b, q_2), (q_2, a, q_1), (q_0, e, q_3), (q_2, a, q_3)\}$. M is non deterministic; $\Delta \subseteq K \times \Sigma \cup \{e\} \times K$.

$$L(M) = \emptyset$$

Q5 $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $\Sigma = \{a, b\}, K = \{q_0, q_1, q_2, q_3\},\ s = q_0, F = \{q_1\}$,
$\Delta = \{(q_0, ab, q_1), (q_1, e, q_0), (q_1, a, q_2), (q_1, ba, q_2), (q_2, a, q_2), (q_0, e, q_3), (q_1, a, q_3)\}$. M is non deterministic; $\Delta \subseteq K \times \Sigma^* \times K$, q_2, q_3 are trap states.

$$L(M) = (ab)^+$$

4 Problems

Problem 1 Let L be a language defines as follows

$$L = \{w \in \{a,b\}^* : between\ any\ two\ a's\ in\ w\ there\ is\ an\ even\ number\ of\ consecutive\ b's.\}.$$

1. Describe a regular expression r such that $\mathcal{L}(r) = L$.

Solution Remark that 0 is an even number, hence $a^* \in L$,

$$r = b^* \cup b^*ab^* \cup b^*(a(bb)^*a)^*b^* = b^*ab^* \cup b*(a(bb)^*a)^*b^*$$

2. Construct a *finite state automata* M, such that $L(M) = L$.

Solution 1

Components of M are:

$$\Sigma = \{a,b\}, K = \{q_0,q_1,q_2,q_3\}, s = q_0, F = \{q_0,q_2,q_3\},$$

$$\Delta = \{(q_0,b,q_0),(q_0,a,q_3),(q_0,a,q_1),(q_1,bb,q_1),(q_1,a,q_2),(q_2,e,q_0),(q_3,b,q_3)\}$$

Some elements of $L(M)$ as defined by the state diagram are:

$$a, aaa, bbb, aaaabbb, bbbaaaa, abba, abbabbbba, abbbbbbabba, ...$$

Solution 2

Components of M are:

$$\Sigma = \{a,b\}, K = \{q_0,q_1,q_2\}, s = q_0, F = \{q_0,q_1,q_2\},$$

$$\Delta = \{(q_0,b,q_0),(q_0,e,q_1),(q_1,bb,q_1),(q_1,a,q_1),(q_1,e,q_2),(q_2,b,q_2)\}$$

Problem 2 Let

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0\},\ s = q_0,\ \Sigma = \{a,b\},\ F = \{q_0\}$ and

$$\Delta = \{(q_0,aba,q_0),(q_0,ab,q_0)\}$$

1. List some elements of $L(M)$.

Solution

$$e, ab, abab, ababa, ababaaba, ...$$

2. Write a regular expression for the language accepted by M.

Solution

$$L = (ab \cup aba)^*$$

Problem 3 We know that for any deterministic finite automaton $M = (K,\ \Sigma,\ s,\ \delta,\ F\ )$ the following is true:

$$e \in L(M) \ \ iff \ \ s \in F.$$

Show that the above is not true for all non-deterministic automata.

Solution Let $M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$ for $K = \{q_0, q_1\}, s = q_0, \Sigma = \emptyset, F = \{q_1\}$, and $\Delta = \{(q_0, e, q_1\}$.

$$L(M) = \{e\} \ \ and \ \ s \notin F.$$

Problem 4 For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_2, q_3\}$ and

$$\Delta = \{(q_0, a, q_1), (q_0, e, q_1), (q_0, b, q_2), (q_1, b, q_3), (q_1, e, q_3), (q_2, b, q_2), (q_2, e, q_3), (q_3, a, q_3)\}$$

Write a regular expression describing $L(M)$).

Solution

$$aa^* \cup a^* \cup aba^* \cup ba^* \cup bb^* \cup bb^*a^*$$

Write 4 steps of the general method of transformation the NDFA M, into an equivalent deterministic M'.

Reminder: $E(q) = \{p \in K : (q, e) \overset{*,M}{\longmapsto} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Solution Step 1:

$$E(q_0) = \{q_0, q_1, q_3\},\ E(q_1) = \{q_1, q_3\},\ E(q_2) = \{q_2, q_3\},\ E(q_3) = \{q_3\}.$$

Solution Step 2:

$$\delta(E(q_0), a) = \delta(\{q_0, q_1, q_3\}, a) = E(q_1) \cup E(q_3) = \{q_1, q_3\} \in F,$$

$$\delta(E(q_0), b) = \delta(\{q_0, q_1, q_3\}, b) = E(q_2) \cup E(q_3) \cup \emptyset = \{q_2, q_3\} \in F,$$

Solution Step 3:

$$\delta(\{q_1, q_3\}, a) = \emptyset \cup E(q_3) = \{q_3\} \in F,$$

$$\delta(\{q_1, q_3\}, b) = E(q_3) \cup \emptyset = \{q_3\} \in F,$$

$$\delta(\{q_2, q_3\}, a) = \emptyset \cup E(q_3) = \{q_3\} \in F,$$

$$\delta(\{q_2, q_3\}, b) = \{q_2\} \cup \emptyset = \{q_2\}$$

Solution Step 4:

$$\delta(\{q_3\}, a) = E(q_3) = \{q_3\} \in F, \quad \delta(\{q_3\}, b) = \emptyset,$$

$$\delta(\emptyset, a) = \emptyset, \quad \delta(\emptyset, b) = \emptyset.$$

End of the construction.