YES/NO questions Circle the correct answer. Write SHORT justification.

1. For any language \( L \subseteq \Sigma^*, \Sigma \neq \emptyset \) there is a deterministic automata \( M \), such that \( L = L(M) \).
   **Justify:** only when \( L \) is regular

2. Any finite language is regular.
   **Justify:** any finite language is a finite union of one element regular languages

3. For any deterministic automata \( M \), \( L(M) = \bigcup \{ R(1, j, n) : q_j \in F \} \), where where \( M \) has \( n \) states with \( s = q_1 \) and \( R(1, j, n) \) is the set of all strings in \( \Sigma^* \) that may drive \( M \) from state initial state to state \( q_j \) without passing through any intermediate state numbered \( n + 1 \) or greater, where \( n \) is the number of states of \( M \).
   **Justify:** basic fact and definition

4. \( \Sigma \) in any Generalized Finite Automaton includes some regular expressions.
   **Justify:** definition of GFA

5. For any finite automata \( M \), there is a regular expression \( r \), such that \( L(M) = r \).
   **Justify:** main theorem

6. Pumping Lemma says that we can always prove that a language is not regular.
   **Justify:** PL gives a certain characterization of infinite regular languages

7. Let \( L \) be a regular language, and \( L_1 \subseteq L \), then \( L_1 \) is regular.
   **Justify:** \( L_1 = \{ a^n b^n : n \geq 0 \} \) is a non-regular subset of regular \( L = a^* b^* \)

8. Let \( L \) be a language. The language \( L^R = \{ w^R : w \in L \} \) is regular.
   **Justify:** \( L^R \) is accepted by finite automata \( M^R \) constructed from \( M \) such that \( L(M) = L \)

9. The class of regular languages is closed with respect to subset relation.
   **Justify:**
   Consider
   \[
   L_1 = \{ a^n b^n : n \in \mathbb{N} \}, \quad L_2 = a^* b^* 
   \]
   \( L_1 \subseteq L_2 \) and \( L_1 \) is a non-regular subset of a regular \( L_2 \)
10. $L$ (over $\Sigma$) is regular, so is the language $L_1 = \{xy : x \in L, y \notin L\}$

Justify:
$L_1 = L(\Sigma^* - L)$ and $L$ regular, hence $(\Sigma^* - L)$ is regular (closure under complement), so is $L_1$ by closure under concatenation

PROBLEMS

QUESTION 1
Using the construction in the proof of theorem

A language is regular iff it is accepted by a finite automata

construct a a finite automata $M$ accepting

$L_1 = L = ((ab)^* \cup (bc)^*)ba$

Solution

M1 components:

$K = \{q_1, q_2\}, \Sigma = \{a, b, c\}, s = q_1, F = \{q_2\},$

$\Delta_{M1} = \{(q_1, ab, q_2)\}$

M2 components:

$K = \{q_3, q_4\}, \Sigma = \{a, b, c\}, s = q_3, F = \{q_4\},$

$\Delta_{M2} = \{(q_2, bc, q_4)\}$

M3 components:

$K = \{q_5, q_6\}, \Sigma = \{a, b, c\}, s = q_5, F = \{q_6\},$

$\Delta_{M3} = \{(q_5, ba, q_6)\}$

2. $M_4, M_5$ such that $L(M_4) = L(M_1)^*, L(M_5) = L(M_2)^*$

Solution

M4 components:

$K = \{q_1, q_2, q_7\}, \Sigma = \{a, b, c\}, s = q_7, F = \{q_2, q_7\},$

$\Delta_{M4} = \{(q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1)\}$
M5 components:

\[ K = \{q_3, q_4, q_8\}, \Sigma = \{a, b, c\}, s = q_8, F = \{q_4, q_8\}, \]
\[ \Delta_{M4} = \{(q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\} \]

3. M6 such that \( L(M5) = L(M4) \cup L(M5) \)

Solution

M5 components:

\[ K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_2, q_4, q_7, q_8\}, \]
\[ \Delta_{M5} = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3)\} \]

4. M = M5M3, i.e M is such that \( L(M) = L(M5)L(M3) \).

M components:

\[ K = \{q_1, q_2, q_3, q_4, q_7, q_8, q_9\}, \Sigma = \{a, b, c\}, s = q_9, F = \{q_6\}, \]
\[ \Delta_{M5} = \Delta_{M4} \cup \{(q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\} \]
\[ = \{(q_9, e, q_7), (q_9, e, q_8), (q_7, e, q_1), (q_1, ab, q_2), (q_2, e, q_1), (q_8, e, q_3), (q_3, bc, q_4), (q_4, e, q_3), \]
\[ (q_7, e, q_5), (q_8, e, q_5), (q_2, e, q_5), (q_4, e, q_5), (q_5, ba, q_6)\} \]

Question 2 Evaluate r, such that

\[ L(r) = L(M) \]

using the Generalized Automata Construction for

\[ M = \{(q_1, q_2), \{a, b\}, s = q_1, \]
\[ \Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, F = \{q_2\}\]  

Step 1: Construct a generalized GM that extends M, i.e. such that \( L(M) = L(GM) \)

Solution

\[ GM = \{(q_1, q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \]
\[ \Delta = \{(q_1, a, q_1), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_1)\}, (q_3, e, q_1), (q_2, e, q_4)\} \]

Step 2: Construct \( GM1 \simeq GM \simeq M \) by elimination of \( q_1 \).

Solution

\[ GM1 = \{(q_2, q_3, q_4), \{a, b\}, s = q_3, F = \{q_4\} \]
\[ \Delta = \{(q_3, a^*b, q_2), (q_2, a, q_2), (q_2, ba^*b, q_2)\}, (q_2, e, q_4)\} \]

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Step 3: Construct $GM_2 \simeq GM_1 \simeq GM \simeq M$ by elimination of $q_2$.

Solution

$$GM_2 = (\{q_3, q_4\}, \{a, b\}, s = q_3, F = \{q_4\})$$
$$\Delta = \{(q_3, a^* b (ba^* b \cup a)^*, q_4)\}$$

Answer: the language is

$$L(M) = a^* b (ba^* b \cup a)^*$$

QUESTION 3

Show that the language

$$L = \{xyx^R : x, y \in \Sigma^*\}$$

is regular for any $\Sigma$.

Solution Take $x = e \in \Sigma^*$. The language

$$L_1 = \{eye^R : e, y \in \Sigma^*\} \subseteq L$$

and $L_1 = \Sigma^*$. We get $\Sigma^* \subseteq L \subseteq \Sigma^*$ and hence $L = \Sigma^*$ is regular.