PART 1: YES/NO QUESTIONS (10 points)

Circle the correct answer. Write SHORT justification.

Answers without justification will not receive credit

1. The set $K$ of states of a deterministic finite automaton can be empty.
   **Justify:** $s \in K$

2. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$.
   **Justify:** element of $K \times \Sigma^*$

3. Given $M = (K, \Sigma, \delta, s, F)$, $L(M) = \{ w \in \Sigma^* : \exists q \in K: (s, w) \vdash^*_M (q, e) \}$.
   **Justify:** only when $q \in F$

4. For any $M = (K, \Sigma, \delta, s, F)$, $L(M) \neq \emptyset$
   **Justify:** for any $M$, such that $F = \emptyset$ or $\Sigma = \emptyset$ we have that $L(M) = \emptyset$.

5. There is a deterministic $M = (K, \Sigma, \delta, s, F)$, such that $e \in L(M)$
   **Justify:** any $M$ such that $s \in F$ as we we proved a Theorem:
   $e \in L(M)$ \iff $s \in F$

6. Given a non-deterministic automaton $M = (K, \Sigma, \Delta, s, F)$ as defined in the book, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds

   \[(q, aw) \vdash_M (q', w) \text{ iff } (q, a, q') \in \Delta \text{ and } a \in \Sigma \cup \{e\} \text{.}\]

   **Justify:** it is the book definition

7. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then $M$ is also non-deterministic, as defined in the lecture.
   **Justify:** $\Sigma \cup \{e\} \subseteq \Sigma^*$.

8. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then $M$ is also non-deterministic.
   **Justify:** any function is a relation and $\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K$.

9. We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a computation of the length $n$ as a sequence

   \[(q_1, w_1), (q_2, w_2), \ldots, (q_n, w_n), \text{ } n \geq 1\]
of configurations, such that \( q_1 = q, \ q_n = q', \ w_1 = w, \ w_n = w' \) and 
\( (q_i, w_i) \vdash^* M (q_{i+1}, w_{i+1}) \) for \( i = 1, 2, ..n - 1 \).

**Statement:** For any \( M \), a computation of the length 1 exists.

**Justify:** TRIVIAL path (case \( n=1 \)) of length one always exists

10. A **trap state** of a DFA automaton \( M \) is any of its states that traps computations of \( M \)

**Justify:** Definition: A **trap state** of a DFA automaton \( M \) is any of its states that does not influence the language \( L(M) \) of \( M \)

**TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA**

**BOOK DEFINITION:** \( M = (K, \Sigma, \Delta, s, F) \) is non-deterministic when

\[
\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K
\]

**Lecture Definition:** \( M = (K, \Sigma, \Delta, s, F) \) is non-deterministic when

\[
\Delta \subseteq K \times \Sigma^* \times K
\]

**SOLVING PROBLEMS** you can use any of these definitions.

**PART 2: Few Very Short Questions** For the automata \( M \) below

1. State and explain whether \( M \) represents a deterministic or a non-deterministic automaton.

2. Write down a regular expression representing \( L(M) \).

**Q1:** \( M_1 \) has components: \( K = \{ q \}, \ s = q, \ \Sigma = \emptyset, \ \delta = \emptyset, \ F = \emptyset \).

**Solution**

1. \( M_1 \) is deterministic.

2. \( L(M_1) = \emptyset \).

**Q2:** \( M_2 \) has components: \( K = \{ q_0, q_1, q_2 \}, \ s = q_0, \ \Sigma = \{ a, b \}, \ \delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}, \ F = \{ q_1 \} \).

1. \( M_2 \) is non-deterministic; \( \delta \) is not a function with the domain \( K \times \Sigma \). It can be completed to a function by adding some trap states. But the trap states information was not it was not stated in the problem.
2. \( L(M2) = aa^* \).

Q3: \( M3 \) \( K = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}, F = \{q_1, q_2\} \).

Solution \( M3 \) is NOT an automaton. It does not have the INITIAL state!

PART 3: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton \( M \) such that

\[ L(M) = \{w \in \{a, b\}^* : \text{neither } bb \text{ nor } aa \text{ is a substring of } w\} \]

Draw a state diagram and specify all components \( K, \Sigma, \delta, s, F \) of \( M \). Justify your construction.

Solution

Components of \( M = (K, \Sigma, \delta, s, F) \) are:

\( \Sigma = \{a, b\}, K = \{q_0, q_1, q_2, q_3\}, s = q_0, q_3 \) is a trap state, \( F = \{q_0, q_1, q_2\} \).

We define \( \delta \) on non-trap states as follows.

\( \delta(q_0, a) = q_1 \), \( \delta(q_0, b) = q_2 \),
\( \delta(q_1, b) = q_2 \),
\( \delta(q_2, a) = q_1 \).

\( M \) accepts strings \( a, aba, ababa \ldots \) or \( b, bab, baba \ldots \) etc and never \( aa, bb, \) etc...

QUESTION 4 Let \( M \) be defined as follows

\( M = (K, \Sigma, s, \Delta, F) \)

for \( K = \{q_0, q_1, q_2, q_3\}, s = q_0 \)
\( \Sigma = \{a, b, c\}, F = \{q_3\} \) and
\( \Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\} \).

1. Find the regular expression describing the \( L(M) \). Simplify it as much as you can. Explain your steps.

Solution \( L(M) = (abc)^*abb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa = (abc)^*(abbb \cup baa) \).

We used the property:

\[ LL_1 \cup LL_2 = L(L_1 \cup L_2). \]

2. Write down (you can draw the diagram) an nondeterministic automata \( M' \) such that \( M' \equiv M \) and \( M' \) is defined by the BOOK definition.
Solution We apply the "stretching" technique to $M$ and the new $M'$ is as follows.

$$M' = (K \cup \{p_1, p_2, \ldots, p_5\}, \Sigma, s = q_0, \Delta', F' = F)$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$

$\Sigma = \{a, b\}$, $F = \{q_3\}$ and

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1),$

$(q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$. 
