CSE303 Q2 SOLUTIONS

PART 1: YES/NO QUESTIONS (10 points)

Circle the correct answer. Write **SHORT justification.**

Answers without justification will not receive credit

1. The set K of states of a deterministic finite automaton can be empty.
   Justify: $s \in K$ **n**
2. A configuration of a deterministic finite automaton $M = (K, \Sigma, \delta, s, F)$ is any element of $K \times \Sigma^* \times K$.
   Justify: element of $K \times \Sigma^*$ **n**
3. Given $M = (K, \Sigma, \delta, s, F)$,
   $L(M) = \{w \in \Sigma^* : \exists q \in K (s, w) \vdash^*_M (q, e)\}$.
   Justify: only when $q \in F$ **n**
4. For any $M = (K, \Sigma, \delta, s, F)$, $L(M) \neq \emptyset$
   Justify: for any M, such that $F = \emptyset$ or $\Sigma = \emptyset$ we have that $L(M) = \emptyset$.
   n
5. There is a deterministic $M = (K, \Sigma, \delta, s, F)$, such that $e \in L(M)$
   Justify: any M such that $s \in F$ as we we proved a **Theorem**: $e \in L(M)$ iff $s \in F$ **y**
6. Given a non-deterministic automaton $M = (K, \Sigma, \Delta, s, F)$ as defined in the book, a binary relation $\vdash_M \subseteq (K \times \Sigma^*) \times (K \times \Sigma^*)$ is a one step computation iff the following condition holds

   $(q, aw) \vdash_M (q', w)$ iff $(q, a, q') \in \Delta$ and $a \in \Sigma \cup \{e\}$.

   Justify: it is the book definition
   y
7. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.
   Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$.
   y
8. If $M = (K, \Sigma, \delta, s, F)$ is a deterministic, then M is also non-deterministic.
   Justify: any function is a relation and $\delta \subseteq K \times \Sigma \times K \subseteq K \times \Sigma \cup \{e\} \times K$ **y**
9. We define, for any (deterministic or non-deterministic $M = (K, \Sigma, \Delta, s, F)$ a computation of the length n from (q, w) to (q', w') as a sequence

   $$(q_1, w_1), (q_2, w_2), ..., (q_n, w_n), \ n \geq 1$$

   of configurations, such that $q_1 = q, \ q_n = q', \ w_1 = w, \ w_n = w'$ and $(q_i, w_i) \vdash^*_M (q_{i+1}, w_{i+1})$ for $i = 1, 2, ..n - 1$.

Statement:For any M, a computation of the length 1 exists.

Justify: TRIVIAL path (case n=1) of length one always exists **y**

10. A **trap state** of a DFA automaton M is any of its states that traps computations of M

    Justify: Definition: A **trap state** of a DFA automaton M is any of its states that does not influence the language L(M) of M **n**

TWO DEFINITIONS OF NON DETERMINISTIC AUTOMATA

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

PART 2: Few Very Short Questions For the automata M below

1. State and explain whether M represents a deterministic or a non-deterministic automaton.

2. Write down a regular expression representing $L(M)$.

Q1: M1 has components: $K = \{q\}$, $s = q, \Sigma = \emptyset, \delta = \emptyset, F = \emptyset$.

Solution

1. $M1$ is deterministic.

2. $L(M1) = \emptyset$.

Q2: M2 has components: $K = \{q_0, q_1, q_2\}$, $s = q_0, \Sigma = \{a, b\}, \delta = \{(q_0, a, q_1), (q_1, a, q_1), (q_0, b, q_2)\}$, $F = \{q_1\}$.

1. $M2$ is non-deterministic; δ is not a function with the domain $K \times \Sigma$. It can be completed to a function by adding some trap states. But the trap states information was not it was not stated in the problem.

2. $L(M2) = aa^*$.

Q3: M3 $K = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \Delta = \{(q_0, a, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_1, b, q_2), (q_2, ab, q_2)\}$, $F = \{q_1, q_2\}$.

Solution $M3$ is NOT an automaton. It does not have the INITIAL state!

PART 3: PROBLEMS

QUESTION 1 Construct a deterministic finite automaton M such that

$$L(M) = \{w \in \{a,b\}^* : \textit{ neither } bb \textit{ nor } aa \textit{ is a substring of } w\}.$$

Draw a state diagram and specify all components K, Σ, δ, s, F of M. Justify your construction.

Solution

Components of $M = (K, \Sigma, \delta, s, F)$ are:
$\Sigma = \{a,b\}$, $K = \{q_0, q_1, q_2, q_3\}$, $s = q_0$, q_3 is a trap state, $F = \{q_0, q_1, q_2\}$.
We define δ on non-trap states as follows.
$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_2$,
$\delta(q_1, b) = q_2$,
$\delta(q_2, a) = q_1$.

M **accepts** strings $a, aba, ababa$.... or $b, bab, baba$.. etc and never aa, bb, etc...

QUESTION 4 Let M be defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2, q_3, \}$, $s = q_0$
$\Sigma = \{a,b,c\}$, $F = \{q_3\}$ and
$\Delta = \{(q_0, abc, q_0), (q_0, ab, q_1), (q_1, bb, q_3), (q_0, b, q_2), (q_2, aa, q_3)\}$.

1. Find the regular expression describing the $L(M)$. Simplify it as much as you can. Explain your steps.

Solution $L(M) = (abc)^*abbb \cup abbb \cup (abc)^*baa \cup ba = (abc)^*abbb \cup (abc)^*baa = (abc)^*(abbb \cup baa)$.

We used the property:

$$LL_1 \cup LL2 = L(L_1 \cup L_2).$$

2. Write down (you can draw the diagram) an nondeterministic automata M' such that $M' \equiv M$ and M' is defined by the BOOK definition.

Solution We apply the "stretching" technique to M and the new M' is is as follows.

$$M' = (K \cup \{p_1, p_2, ..., p_5\}\ \Sigma,\ s = q_0,\ \Delta',\ F' = F\ )$$

for $K = \{q_0, q_1, q_2\}$, $s = q_0$
$\Sigma = \{a, b\}$, $F = \{q_3\}$ and

$\Delta' = \{(q_0, b, q_2), (q_0, a, p_1), (p_1, b, p_2), (p_2, c, q_0), (q_0, a, p_3), (p_3, b, q_1),$
$(q_1, b, p_4), (p_4, b, q_3), (q_0, b, q_2), (q_2, a, p_5), (p_5, a, q_3)\}$.