CSE303 NEW MIDTERM SOLUTIONS Spring 2012

1 YES/NO questions

Circle the correct answer (each question is worth 2pt.) Write SHORT justification.

1. For any function f from $A \neq \emptyset$ onto A, f has property

$$\forall a \in A \; \exists b \in A(f(b) = a).$$

   Justify: definition of "onto" function.

   y

2. Some infinite sets have the same cardinality.

   Justify: $|N| = |2N|$ and N (natural numbers) and $2N$ (even numbers) are infinite sets. **y**

3. $\{\{a, b\}\} \in \{a, b, \{a, b\}\}$
   Justify: $\{\{a, b\}\} \subseteq \{a, b, \{a, b\}\}$ as $\{a, b\} \in \{a, b, \{a, b\}\}$

   n

4. For any function $R \subseteq A \times A$, R^{-1} exists.

   Justify: *Theorem:* The inverse function R^{-1} exists iff R is $1-1$ and "onto".

   n

5. A language L is regular iff $L = \mathcal{L}(r)$ for some $r \in \Sigma^*$.

   Justify: only when r is a regular expression.

   n

6. $L^+ = L^* - \{e\}$.

   Justify: only when $e \notin L$. When $e \in L$ we get that $e \in L^+$ and $e \notin L^* - \{e\}$.

   n

7. For any languages L_1, L_2, $(L_1 \cup L_2) \cup L_1 = L_2$.
   Justify: languages are sets, so it holds only when only when $L_1 \subseteq L_2$.

   n

8. $(\emptyset^* \cap b^*) \cup \emptyset^*$ describes a language with two elements.
   Justify: the set $(\{e\} \cap \{b\}^*) \cup \{e\} = \{e\} \cup \{e\} = \{e\}$ has one element. **n**

9. For any automata M, $L(M) = \emptyset$ iff the set F of its final states is empty.

   Justify: Let M be such that $\Sigma = \emptyset, F \neq \emptyset, s \notin F$, we get $L(M) = \emptyset$. **n**

10. If $M = (K, \Sigma, \Delta, s, F)$ is a non-deterministic as defined in the book, then M is also non-deterministic, as defined in the lecture.

    Justify: $\Sigma \cup \{e\} \subseteq \Sigma^*$ **y**

11. Let M be a finite state automaton, $L(M) = \bigcup_{q \in F}\{w \in \Sigma^* : (s, w) \overset{*,M}{\longmapsto} (q, e)\}$.

    Justify: $w \in \bigcup_{g \in F}\{w \in \Sigma^* : (s, w) \overset{*,M}{\longmapsto} (q, e)\}$ iff there is $q \in F$ such that $(s, w) \overset{*,M}{\longmapsto} (q, e)$ iff $w \in L$. **y**

12. For any finite automats M_1, M_2 , $L(M_1) = L(M_2)$ iff $M_1 \equiv M_2$.
    Justify: definition of automata equivalency. **y**

13. DFA and NDFA recognize the same class of languages.

    Justify: theorem proved in class **y**

2 Two definitions of a non-deterministic automaton

BOOK DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when

$$\Delta \subseteq K \times (\Sigma \cup \{e\}) \times K$$

OBSERVE that Δ is always finite because K, Σ are finite sets.

LECTURE DEFINITION: $M = (K, \Sigma, \Delta, s, F)$ is non-deterministic when Δ is finite and

$$\Delta \subseteq K \times \Sigma^* \times K$$

OBSERVE that we have to say in this case that Δ is finite because Σ^* is infinite.

SOLVING PROBLEMS you can use any of these definitions.

3 PROBLEMS

PROBLEM 1 Let $\Sigma = \{a, b\}$. Show that

$$(a \cup b)^* a (a \cup b)^* = \Sigma^* a \Sigma^*.$$

Solution Observe that

$$\mathcal{L}(a \cup b)^* = (\{a\} \cup \{b\})^* = \{a, b\}^* = \Sigma^*.$$

Hence $(a \cup b)^* a (a \cup b)^* = \Sigma^* a \Sigma^*$.

PROBLEM 2 Write a regular expression r, such that $L = \mathcal{L}(r)$ for L over $\Sigma = \{a, b\}$ defined as

$$L = \{w \in \Sigma^* : \ w \text{ has no more than three } a's\}.$$

Solution

$$r = b^* \cup b^*ab^* \cup b^*ab^*ab^* \cup b^*ab^*ab^*ab^*$$

PROBLEM 3 Let L be a language defines as follows

$$L = \{w \in \{a,b\}^* : \textit{between any two } a's \textit{ in } w \textit{ there is an even number of consequitive } b's.\}.$$

1. Describe a regular expression r such that $\mathcal{L}(r) = L$ (Meaning of r is L).

Solution Remark that 0 is an even number, hence $a^* \in L$,

$$r = b^*a^*b^* \cup b^*(a(bb)^*a)^*b^* = (b^*a(bb)^*ab^*)^*$$

2. Construct a *finite state automata* M, such that $L(M) = L$.

Solution

Components of M are:

$$\Sigma = \{a,b\}, K = \{q_0, q_1, q_2, q_3\}, s = q_0, F = \{q_0, q_2, q_3\},$$

$$\Delta = \{(q_0,b,q_0), (q_0,a,q_3), (q_0,a,q_1), (q_1,bb,q_1), (q_1,a,q_2), (q_2,e,q_0), (q_2,b,q_2), (q_3,b,q_3), (q_3,e,q_0)\}$$

Some elements of $L(M)$ as defined by the state diagram are:

$$b, a, aaaa, aabbb, bbbbaaaa, abbb, abba, bbbabbabbb, abbabbbabba, abbaabbaabba, ...$$

PROBLEM 4 Let

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0\},\ s = q_0,\ \Sigma = \{a,b\},\ F = \{q_0\}$ and

$$\Delta = \{(q_0, aba, q_0), (q_0, ab, q_0)\}$$

1. List some elements of $L(M)$.

Solution

$$e, ab, abab, ababa, ababaaba, ...$$

2. Write a regular expression for the language accepted by M.

Solution

$$L = (ab \cup aba)^*$$

3. Use the Book Definition to define an automaton M' such that $M' \equiv M$ (use the "STRETCH" technique).

$K' = K \cup \{p_1, p_2, p_3\}, \Delta' = \Delta_{\Sigma\cup e} \cup \{(g_0, a, p_1), (p_1, b, p_2), (p_2, a, q_0), (q_0, b, p_3), (p_3, b, q_0)\}$

where $\Delta_{\Sigma\cup e}$ denes those elements of Δ that involve only elements of $\Sigma \cup e$.

PROBLEM 5 (20pts)

For M defined as follows

$$M = (K,\ \Sigma,\ s,\ \Delta,\ F\ )$$

for $K = \{q_0, q_1, q_2\},\ s = q_0$
$\Sigma = \{a, b\},\ F = \{q_1, q_2\}$ and

$$\Delta = \{(q_0, ab, q_1), (q_0, e, q_1), (q_0, b, q_2), (q_1, a, q_1), (q_2, bb, q_2), (q_1, e, q_2)\}$$

Write a regular expression describing $L(M)$).

$$aba^*(bb)^8 \cup a^*(bb)^* \cup b(bb)^*$$

Write 5 steps of the general method of transformation the NDFA M, into an equivalent deterministic M'.

Reminder 1: $E(q) = \{p \in K : (q, e) \overset{*,M}{\longmapsto} (p, e)\}$ and

$$\delta(Q, \sigma) = \bigcup\{E(p) : \exists q \in Q,\ (q, \sigma, p) \in \Delta\}.$$

Reminder 2: The above definitions apply to the Book definition of non-deterministic automata.

The proper DIAGRAM of **new** (book definition - use "stretch" method) M is:

$K' = K \cup \{p_1, p_2\}, \Delta' = \Delta_{\Sigma\cup e} \cup \{(g_0, a, p_1), (p_1, b, q_1), (q_2, b, p_2), (p_2, b, q_2)\}$

where $\Delta_{\Sigma\cup e}$ denes those elements of Δ that involve only elements of $\Sigma \cup e$.

Solution: apply definition to M' defined above.