CSE 526, Spring 2020: HW2

Part a due: Tue., March 3. **Part b** due: Tue Mar 10.

Instructions:

- Part (a) should contain:
  - Complete formal proof of Q1.
  - Detailed sketch of all other parts, containing sufficient information about the main proof steps for us to give useful feedback.
- Part(b) should contain formal answers to all questions.
- For each part, submit PDF files with your solution. Latex or otherwise typeset proofs and proof-sketches are preferred. You may also submit scans of handwritten solutions, as long as they are legible.

Q1. Let S be a set, and R be a binary relation over S (i.e., $\subseteq S \times S$). Consider the following three definitions, all attempting to define the *reflexive transitive closure* of R:

R_I^*: The *inductive* transitive closure of R, denoted by R_I^* is the *smallest* set such that:

1. if $s_1 \in S$ ~~and~~ *then* $(s_1, s_1) \in R_I^*$.
2. if $(s_1, s_2) \in R$ and $(s_2, s_3) \in R_I^*$, then $(s_1, s_3) \in R_I^*$.

R_C^* : The *constructive* transitive closure of R, denoted by R_C^* is the union of all T_i, $i \geq 0$ (i.e. $R_C^* = \bigcup_{i \geq 0} T_i$) , where:

$$
\begin{array}{rcl}
T_0 & = & \emptyset \\
T_{i+1} & = & \begin{cases} & \{(s,s) \mid s \in S\} \\ \cup & \{(s_1, s_3) \mid \exists (s_1, s_2) \in R, (s_2, s_3) \in T_i\} \end{cases}
\end{array}
$$

Q1. Show that $R_I^* = R_C^*$.

Q2. Consider the language $\mathbf{B_N}$ of Boolean expressions from the text whose syntax and single-step operational semantics are given below.

Terms and Values:

$$
\begin{array}{rcll}
t & ::= & & \boxed{\textit{Terms:}} \\
 & & \texttt{true} & \\
 & | & \texttt{false} & \\
 & | & \texttt{nor}(t, t) & \\
v & ::= & & \boxed{\textit{Values:}} \\
 & & \texttt{true} & \\
 & | & \texttt{false} &
\end{array}
$$

Evaluation Rules:

$$\texttt{nor}(\texttt{true}, t_2) \rightarrow \texttt{false} \qquad \text{E-NorTrue}$$

$$\texttt{nor}(\texttt{false}, \texttt{false}) \rightarrow \texttt{true} \qquad \text{E-NorFalseFalse}$$

$$\texttt{nor}(\texttt{false}, \texttt{true}) \rightarrow \texttt{false} \qquad \text{E-NorFalseTrue}$$

$$\frac{t_2 \rightarrow t_2'}{\texttt{nor}(\texttt{false}, t_2) \rightarrow \texttt{nor}(\texttt{false}, t_2')} \qquad \text{E-NorFalse}$$

$$\frac{t_1 \rightarrow t_1'}{\texttt{nor}(t_1, t_2) \rightarrow \texttt{nor}(t_1', t_2)} \qquad \text{E-Nor}$$

Q2. (a) Does determinacy hold for $\mathbf{B_N}$? That is, for all t, t', t'', if $t \rightarrow t'$ and $t \rightarrow t''$ then $t' = t''$? Justify.

(b) Does uniqueness of normal forms hold in $\mathbf{B_N}$? That is, for all t, v, v', if $t \rightarrow^* v$ and $t \rightarrow^* v'$ then $v = v'$? Justify.

(c) Does termination hold in $\mathbf{B_N}$? Justify.

For all three parts above, your justification should be as follows. If the property holds, give a formal proof. If the property does not hold give a completely-specified counter example, and explain how it is a counter example.

Errata:

- Feb 29, 9pm: Word "**and**" in the first item in the definition of R_I^* in Q1 was in error. This has been changed to "**then**".

Last Updated: 02/29/2020 at 9:24pm