CSE-526 Principles of Programming Languages

Spring '12

Mid-term Exam

March 13, 2012.

Duration: 1 hour, 20 minutes.

Name: ______________________ **USB ID Number:** ______________

INSTRUCTIONS

Read the following carefully before answering any question.

- Make sure you have filled in your name and USB ID number in the space above.
- Write your answers in the space provided; Keep your answers brief and precise.
- The exam consists of 5 questions, in 9 pages (including this page) for a total of **40** points.

  Question 1 has two pages.

  Question 2 has two pages.

  Question 3 is on a single page.

  Question 4 is on a single page.

  Question 5 has two pages.

GOOD LUCK!

Question	**Max.**	**Score**
1.	9	
2.	8	
3.	5	
4.	6	
5.	12	
Total:	40	

1. [9 points] Recall the language **B** of Boolean expressions from the text (Figure 3-1, page 34). Consider *adding* to **B**, terms and evaluation rules described below, to give a new language **BF1**:

Terms:	***Evaluation Rules:***	
t ::= ... all terms in **B**	$\texttt{and}(t_1, t_2) \to \texttt{if}(t_1, t_2, \texttt{false})$	E-And
$\mid$ $\texttt{and}(t, t)$	$\texttt{not}(t_1) \to \texttt{if}(t_1, \texttt{false}, \texttt{true})$	E-Not
$\mid$ $\texttt{not}(t)$		

There are three parts to this question, below. In each part, you are asked to say whether or not a stated property holds in **BF1** and justify your claim. Your justification should be as follows. If the property holds, and the proof in the book still applies, state "book proof applies". If the property holds and the book proof is no longer valid, give a brief sketch of your proof. If the property does not hold give a counter example.

(a) [3 points] Is the Determinacy Theorem valid for **BF1**? That is, for all t, t', t'', if $t \to t'$ and $t \to t''$ then $t' = t''$? Justify.

(b) [2 points] Is the Uniqueness of Normal Forms Theorem valid in **BF1**? That is, for all t, v, v', if $t \to^* v$ and $t \to^* v'$ then $v = v'$? Justify.

(c) [3 points] Does the Termination Theorem hold in **BF1**? That is, for all t there exists t' in normal form such that $t \to^* t'$? Justify.

2. [8 points] Consider implementing the semantics of **B** (Figure 3-1, page 34) in OCAML, as follows. Represent terms using the datatype:

```
type term = True
          | False
          | If of term * term * term
```

Consider the function

```
let isvalue = function
       True -> true
     | False -> true
     | _ -> false
```

Note that `isvalue` returns `true` if, and only if, the given term is a *value* according to **B**'s semantics.

(a) [3 points] Write an OCAML function `singlestep`: `term` $\rightarrow$ `term` such that `singlestep`(t_1) returns t_2 if, and only if, $t_1 \rightarrow t_2$ in **B**'s single step semantics.

(b) [1 point] State the key property of **B** that enables us to write the OCAML function `singlestep` to faithfully implement `B`'s semantics.

(c) [3 points] Write an OCAML function `reducesto: term` $\rightarrow$ `term` such that if `reducesto`(t_1) returns t_2, then:

- `isvalue`(t_2) is true, and
- $t_1 \rightarrow^* t_2$ according to **B**'s semantics.

(d) [1 point] State the key property of **B** that enables us to write the OCAML function `reducesto` to faithfully implement `B`'s semantics.

3. [5 points] Consider *changing* the language **B** such that, in a term of the form `if`(t_1, t_2, t_3), t_3 is always evaluated to a normal form first, then t_2 is evaluated to a normal form, then t_1 is evaluated to a normal form, and finally, the `if` term is evaluated. Write single-step evaluation rules that encode the above evaluation scheme.

4. [6 points] Consider the *addition* of the following evaluation rule to **NB**:

$$\texttt{succ}(\texttt{pred}(t)) \rightarrow t \qquad \text{E-SuccPred}$$

(a) [3 points] Is the Determinacy Theorem valid in the modified language? That is, for all t, t', t'', if $t \rightarrow t'$ and $t \rightarrow t''$ then $t' = t''$? Justify.

(b) [3 points] Is the Uniqueness of Normal Forms Theorem valid in the modified language? That is, for all t, v, v', if $t \rightarrow^* v$ and $t \rightarrow^* v'$ then $v = v'$? Justify.

For both parts above, your justification should be as follows. If the property holds, and the proof in the book still applies, state "book proof applies". If the property holds and the book proof is no longer valid, give a brief sketch of your proof. If the property does not hold give a counter example.

5. [12 points] This question pertains to the language of untyped lambda calculus (Chapter 5) and call-by-value semantics (Figure 5-3, page 72).

   (a) [1 point] Give an example of a lambda term whose free variable and bound variable sets are disjoint.

   (b) [1 point] Give an example of a lambda term whose free variable and bound variable sets are identical.

   (c) [2 points] Let t_1 and t_2 be two alpha-equivalent lambda terms. Then state the relationship that is always true between their free variable and bound variable sets. (e.g. relationships of the form $bv(t_1) = bv(t_2), fv(t_1) = fv(t_2)$, etc.).

   (d) [2 points] Give lambda expressions t_1 and t_2 such that

   - $t_1 \to t_2$ under call-by-value semantics, and
   - $fv(t_2) \neq fv(t_1)$.

(e) [3 points] What does (($(\lambda x.\ \lambda y.\ y\ x)$ $(\lambda u.\ \lambda v.\ u)$) $(\lambda z.\ z\ z)$) evaluate to, *in one step*, under call-by-value semantics? Show the derivation.

(f) [3 points] What is the normal form of (($(\lambda x.\ \lambda y.\ y\ x)$ $(\lambda u.\ \lambda v.\ u)$) $(\lambda z.\ z\ z)$)? Show the evaluation sequence under call-by-value semantics.

END OF EXAM