CSE 526: Mid-Term Exam

March 22, 2011 | Duration: 1hr 20m | Max: 40 points

Instructions: Please write answers in the plain sheets provided. Please write your name, legibly, on each sheet. Make sure to clearly state the question number for each answer.

1. [15 points] Consider the following definition of relation R^*, specified in the form of inference rules.

$$\frac{s\ R\ t}{s\ R^*\ s}\ \textsc{Refl-1}$$

$$\frac{s\ R\ t}{t\ R^*\ t}\ \textsc{Refl-2}$$

$$\frac{s\ R\ t}{s\ R^*\ t}\ \textsc{Incl}$$

$$\frac{s\ R^*\ t \qquad tR\ u}{s\ R^* u}\ \textsc{Trans-1}$$

Recall the definition of a relation R's reflexive transitive closure as the smallest reflexive and transitive relation that contains R.

(a) [5 points] Is R^* defined using the inference rules above the reflexive transitive closure of R? If so, give the main steps used in proving this. If not, give a counter example.

(b) [5 points] Consider replacing the Trans-1 rule with the following rule:

$$\frac{s\ R\ t \qquad tR\ u}{s\ R^* u}\ \textsc{Trans-2}$$

Is R^* defined using the modified inference rules (i.e. Refl-1, Refl-2, Incl, Trans-2) the reflexive transitive closure of R? If so, give the main steps used in proving this. If not, give a counter example.

(c) [5 points] Now consider replacing the Trans-1 rule with the following rule:

$$\frac{s\ R^*\ t \qquad tR^*\ u}{s\ R^* u}\ \textsc{Trans-3}$$

Is R^* defined using the modified inference rules (i.e. Refl-1, Refl-2, Incl, Trans-3) the reflexive transitive closure of R? If so, give the main steps used in proving this. If not, give a counter example.

2. [7 points] Recall the language **B** of Boolean expressions from the text whose syntax and single-step operational semantics is given below.

Terms and Values:

$$
\begin{array}{rcll}
t & ::= & & \boxed{\textit{Terms:}} \\
 & & \texttt{true} & \\
 & | & \texttt{false} & \\
 & | & \texttt{if}(t, t, t) & \\
v & ::= & & \boxed{\textit{Values:}} \\
 & & \texttt{true} & \\
 & | & \texttt{false} &
\end{array}
$$

Evaluation Rules:

$$\texttt{if}(\texttt{true}, t_2, t_3) \to t_2 \quad \text{E-IfTrue}$$

$$\texttt{if}(\texttt{false}, t_2, t_3) \to t_3 \quad \text{E-IfFalse}$$

$$\frac{t_1 \to t_1'}{\texttt{if}(t_1, t_2, t_3) \to \texttt{if}(t_1', t_2, t_3)} \quad \text{E-If}$$

Consider a change to the semantics where E-IfFalse rule is changed as follows:

$$\texttt{if}(\texttt{false}, t_2, t_3) \to \texttt{if}(\texttt{true}, t_3, t_2) \quad \text{E-IfFalse}$$

(a) [2 points] Does determinacy hold in the modified semantics? That is, for all t, t', t'', if $t \to t'$ and $t \to t''$ then $t' = t''$? Justify.

(b) [2 points] Does uniqueness of normal forms still hold? That is, for all t, v, v', if $t \to^* v'$ and $t \to^* v'$ then $v = v'$? Justify.

(c) [3 points] Does termination still hold? Justify.

For all three parts above, your justification should be as follows. If the property holds, and the proof in the book still applies, state "book proof applies". If the property holds and the book proof is no longer valid, give a brief sketch of your proof. If the property does not hold give a counter example.

3. [18 points] Recall untyped lambda calculus with call-by-value evaluation from the text whose syntax and single-step operational semantics is given below.

Terms and Values:

t	$::=$		***Terms:***
		x	Variables
	$\mid$	$t\ t$	Application
	$\mid$	$\lambda x.\ t$	Abstraction
v	$::=$		***Values:***
	$\mid$	$\lambda x.\ t$	

Evaluation Rules:

$$\frac{t_1 \to t_1'}{t_1\ t_2 \to t_1'\ t_2} \quad \text{E-APP1}$$

$$\frac{t_2 \to t_2'}{v_1\ t_2 \to v_1\ t_2'} \quad \text{E-APP2}$$

$$(\lambda x.\ t_1)\ v_2 \to [x \mapsto v_2]t_1 \quad \text{E-APPABS}$$

(a) [1 point] Give an example of a lambda term which has no free variables.

(b) [1 point] Give an example of a lambda term which has no bound variables.

(c) [1 point] Give an example of a lambda term whose free variable and bound variable sets are disjoint.

(d) [1 point] Give an example of a lambda term whose free variable and bound variable sets are identical.

(e) [2 points] Let t_1 and t_2 be two alpha-equivalent lambda terms. Then state the relationship that is always true between their free variable and bound variable sets. (e.g. relationships of the form $bv(t_1) = bv(t_2), fv(t_1) = fv(t_2)$, etc.).

(f) [3 points] What does $(\ (\ (\lambda x.\ \lambda y.\ x\ y) \quad (\lambda z.\ z)\ ) \quad (\lambda u.\ \lambda v.\ u)\ )$ evaluate to, *in one step*? Show the derivation.

(g) [3 points] What is the normal form of $(\ (\ (\lambda x.\ \lambda y.\ x\ y) \quad (\lambda z.\ z)\ ) \quad (\lambda u.\ \lambda v.\ u)\ )$? Show the evaluation sequence.

(h) [6 points] Consider the claim that:

If t_1 is a sub-term of t_2, and t_2 is in normal form, then t_1 is in normal form.

Is the above claim true? If so, give a sketch of the proof. If not, give a counter-example.

END OF EXAM