

# CSE 526: Principles of Programming Languages

Duration: 30m

Mid-Term Make-Up Test: April 15, 2010

Max: 30 points

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1. [2 points each] Determine the type of each of the following terms in typed lambda calculus with simple extensions. For each term, write its type in the space provided, or state that it is not well-typed. *There is no need to show a type derivation.*

(a)  $\lambda x : A \rightarrow B. \lambda y : B. x$

(b)  $\lambda x : A \rightarrow B. \lambda y : B. x y$

(c)  $\lambda x : A. \text{let } y = \lambda z : A. z \text{ in } (y x)$

(d)  $\lambda x : A. \lambda y : A \rightarrow B. \text{let } z = (y x) \text{ in } (y z)$

(e)  $\lambda x : \{p : A, q : \{r : A, s : A \rightarrow B\}\}. (x.q.s \ x.p)$

(f)  $\text{fix } \lambda x : A. \lambda y : B. x$

(g)  $\lambda x : \text{Ref Ref } A. !x$

(h)  $\lambda x : \text{Ref Ref } A. \lambda y : A. !x := y$

(i)  $\lambda x : \text{Ref } A. \text{ref } x$

(j)  $\lambda x : A. \lambda y : \text{Ref Ref } A. y := x$

2. Recall language **B** from the text whose syntax and single-step operational semantics is given below.

<i>Terms and Values:</i>	<i>Evaluation Rules:</i>
$t ::=$ <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-bottom: 5px;"><b>Terms:</b></div>             true              false              if(<math>t, t</math>)         </div>	$\text{if}(\text{true}, t_2, t_3) \rightarrow t_2 \quad \text{E-IFTRUE}$ $\text{if}(\text{false}, t_2, t_3) \rightarrow t_3 \quad \text{E-IFFALSE}$
$v ::=$ <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-bottom: 5px;"><b>Values:</b></div>             true              false         </div>	$\frac{t_1 \rightarrow t'_1}{\text{if}(t_1, t_2, t_3) \rightarrow \text{if}(t'_1, t_2, t_3)} \quad \text{E-IF}$

(a) [3 points] What does the following term evaluate to, *in one step*? Show the derivation.

$\text{if}(\text{if}(\text{true}, \text{false}, \text{true}), \text{false}, \text{if}(\text{false}, \text{false}, \text{false}))$

(b) [3 points] What is the normal form of the following term? Show the evaluation sequence.

$\text{if}(\text{if}(\text{true}, \text{false}, \text{true}), \text{false}, \text{if}(\text{false}, \text{false}, \text{false}))$

(c) [2 points] Consider a theorem which states that *if  $t$  is a normal form then  $t$  is a value*. What form of induction will you use to prove this theorem? Justify your choice (be very brief).

(d) [2 points] Consider the determinacy theorem which states that *if  $t \rightarrow t'$  and  $t \rightarrow t''$  then  $t' = t''$* . Which form of induction will you use to prove this theorem? Justify your choice (be very brief).