# CSE 526: Spring 2012
## Untyped Lambda Calculus

## Example problems

For each of the following lambda expressions, (a) say what it will evaluate to in one step using call-by-value semantics, and (b) what is its normal form.

1. $(\lambda x.\ \lambda y.\ y\ x)\ (\lambda w.\ w)\ (\lambda x.\ x\ x)$

2. $(\lambda x.\ \lambda y.\ y\ y\ x)\ (\lambda w.\ w)\ (\lambda x.\ x\ x)$

3. $(\lambda x.\ \lambda y.\ \lambda z.\ y\ z\ x)\ (\lambda w.\ w)\ (\lambda x.\ \lambda y.\ y)\ (\lambda x.\ x\ x)$

4. $\lambda x.\ \lambda y.\ y\ x$

5. $\lambda x.\ \lambda y.\ x\ y$

6. (plus $c_2$ $c_1$) where $c_1$ and $c_2$ are Church numerals for 1 and 2, and plus is the lambda expression for plus (page 61 of text book).

7. (plus $c_1$) where $c_1$ is the Church numeral for 1, and plus is the lambda expression for plus (page 61 of text book).

8. You can construct additional exercises by using the Church booleans, numerals and encodings of boolean and arithmetic functions given in section 5.2 of the book.

# Solutions

1. (a) First step: $(\lambda y.\ y\ (\lambda w.\ w))\ (\lambda x.\ x\ x)$

   (b) Normal form: $(\lambda w.\ w)$

2. (a) First step: $(\lambda y.\ y\ y\ (\lambda w.\ w))\ (\lambda x.\ x\ x)$

   (b) Normal form: None

3. (a) First step: $(\lambda y.\ \lambda z.\ y\ z\ (\lambda w.\ w))\ (\lambda x.\ \lambda y.\ y)\ (\lambda x.\ x\ x)$

   (b) Normal form: $\lambda w.\ w$

4. (a) No step

   (b) Already in N.F.

5. (a) No step

   (b) Already in N.F.

6. (a) First step: $(\lambda n.\ \lambda s.\ \lambda z.\ \mathsf{c}_2\ s\ (n\ s\ z))\ \mathsf{c}_1$

   (b) Normal form: $\lambda s.\ \lambda z.\mathsf{c}_2\ s\ (\mathsf{c}_1\ s\ z)$

7. (a) First step: $\lambda n.\ \lambda s.\ \lambda z.\ \mathsf{c}_1\ s\ (n\ s\ z))$

   (b) Normal form: same as above; lambda abstractions are values in the value-passing semantics.