An Efficient 2-D Jacobian Iteration Modeling for Image Interpolation

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***Abstract*—This paper proposes a new interpolation approach for obtaining high resolution (HR) images from its low resolution (LR) images. We are using the Least Squared based block by block prediction scheme to estimate the predictors using Jacobian iteration method. In spite of Jacobian's Iterative property of convergence for diagonally dominant matrices only, our proposed method uses this property effectively for all types of matrices, and found a set of prediction coefficients using a small number of iterative steps. Due to its lesser computational cost it can be used in real time applications too. Use of iterative methods like Jacobi gives an advantage of its application over images which gives singular matrices during operation. Experimental results indicates that the proposed algorithm gives better quantitative performance as compared to other conventional interpolation techniques.**

Keywords: Convergence, Interpolation, Iteration, Jacobian, Parameters.

I. Introduction

Interpolation is a technique that pervades many applications. It enables us to get a high resolution image from its low resolution version. Image interpolation is practiced in improved definition television (IDTV) receiver design, photograph zooming and remote sensing. Besides this, it is also applied in medical imaging, computer graphics, satellite imagery and in various other fields.

Conventional interpolation methods use Nearest Neighbor, Bilinear, Bicubic, Spline etc. interpolation algorithms. In case of Nearest Neighbor method, the value of a new pixel is taken as the translated value nearest to it. In bilinear method, the interpolated value is the weighted average (0.25 in case of 4 neighboring pixels used for interpolation) of the one translated values on either side. Whereas, Bicubic method uses the interpolated values as the weighted average of two translated values on either side. But, they fail to work properly near edge structures. However, they are used in many applications due to their less computational complexity.

In order to preserve the edge structures in an image, various interpolation algorithms have been developed so far. These entire Edge Preserved algorithms are highly complex as it requires estimation of covariance matrix with involvement of matrix inversion. Li and Orchard [4] suggested the edge directed interpolation algorithm, in which the missing pixels are interpolated based on the estimated covariance of the HR image from the covariance of LR image (NEDI) which involves lot of computational power. Ketan and Oscar[6] also suggested an autoregressive method using Gauss-Seidel optimization relying on both LR and HR pixels. Jaiswal and Jakhetiya [7] have suggested an algorithm based on down sampling of image and then using least square estimation which consumes high computational power. Jaiswal and Kumar[5] have also suggested an algorithm which is less complex and uses fixed set of predictors but these predictors being fixed dont adapt so well on all set of images. Oscar and Chan[2] suggested a content adaptive interpolation scheme which is also computationally simple but again fails to gives good quality result.

The main contribution of this work is to develop a low complex image interpolation technique with a better quality compared to competitive interpolation algorithms reported in literature. For this purpose, we divide the Low Resolution(LR) image into a set of non-overlapping blocks. For each block, we found a set of prediction coefficients using a small number of Jacobian iterations. Each non-overlapping block has a set of prediction coefficients which results in good quality besides the method being computationally simple.

Remaining part of the paper is organized in sections and subsections as follows. Section II gives an overview of Jacobian Iteration Scheme [7]. Section III discusses proposed algorithm which includes the division of images in various non-overlapping blocks to get covariance matrix and estimation of prediction coefficients. Simulation results and concluding remarks are made in section IV and V respectively.

II. Overview of Jacobian Iteration Method

Consider a linear system $Ax = B$

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3}... & ...A_{1,n} \\ A_{2,1} & A_{2,2} & A_{2,3}... & ...A_{2,n} \\ A_{3,1} & A_{3,2} & A_{3,3}... & ...A_{3,n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ A_{n,1} & A_{n,2} & A_{n,3}... & ...A_{n,n} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ . \\ . \\ . \\ B_n \end{bmatrix}$$

where $\mathrm{A}_{i,j}, x_i, B_i \in R, (i, j = 1, ..., n)$.

The Jacobi method is the simplest method to solve a linear system [9].

For every equation, $A_{i,1}x_1 + A_{i,2}x_2 + ... + A_{i,n}x_n = B_i$ of $Ax = B$,

$A = D_z + R_z$ where,

$$D_z = \begin{bmatrix} A_{1,1} & 0 & 0... & ...0 \\ 0 & A_{2,2} & 0... & ...0 \\ 0 & 0 & A_{3,3}... & ...0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & 0... & ...A_{n,n} \end{bmatrix}$$

and

$$R_z = \begin{bmatrix} 0 & A_{1,2} & A_{1,3}... & ...A_{1,n} \\ A_{2,1} & 0 & A_{2,3}... & ...A_{2,n} \\ A_{3,1} & A_{3,2} & 0... & ...A_{3,n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ A_{n,1} & A_{n,2} & A_{n,3}... & ...0 \end{bmatrix}$$

The solution is then obtained iteratively by

$$x^{(k+1)} = D_z^{-1}(B - \Sigma R_z x^{(k)}) \qquad (1 \leq i \leq n).$$

The component-wise form of the Jacobi method is :

$$x_i{}^{(k+1)} = (B_i - \Sigma A_{i,j} x_j{}^{(k)})/A_{i,i}. \qquad i \neq j \qquad (1)$$

The computation of $x_i^{(k+1)}$ requires each element in $x^{(k)}$ except itself. In case of Jacobi iteration, the iterative steps first leads to inexact result and subsequently refines its result at each iteration step with residual converging at higher rate. But this happens only when the matrix A is diagonally dominant i.e.

$$|A_{i,i}| \geq \Sigma |A_{i,j}|, \qquad (2)$$

where $i \neq j$. Otherwise with each iterative steps the residual diverges.

Since the set of matrices encountered during processing of various kind of images don't follow the property of diagonally dominance, thus the iterative methods like Jacobi or Gauss-Seidel can't be used effectively as they get diverged .

III. Proposed Algorithm

In this section, we outline the overall interpolation algorithm based on Jacobian Iteration. The algorithm is basically divided into two phases although the unknown pixels are categorized in three categories based on their spatial location. In this algorithm, source image S (LR) of size $M \times N$ is expanded to interpolated image I (HR) of size $2M \times 2N$. The expansion from source image to interpolated image follows the mapping:

$$M : S \rightarrow I$$

according to the equation

$$M(S(i,j)) = I(2i-1, 2j-1). \qquad (3)$$

After this mapping, the remaining $(\frac{3}{4})^{th}$ pixels of I, are predicted in two phases. The remaining pixels are located at even-even, odd-even and even-odd spatial positions represented in gray and white dots respectively.

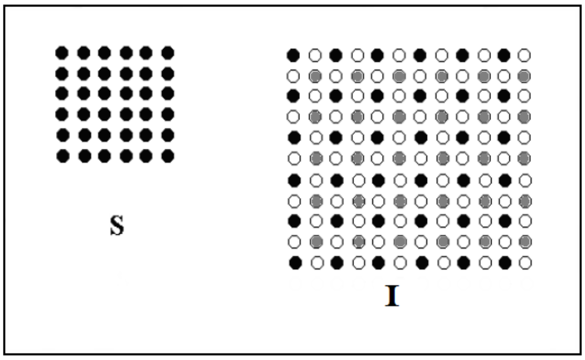

Fig. 1. First stage expansion of an image from Low Resolution to High Resolution (Black dots in image I are original pixels from LR image).

A. Phase-I

In this phase, we predict the missing pixels lying at even-even position of spatial coordinates of Image I of size 512×512. The unfilled pixels at even-even position can be filled as follows:

1) We divide the image I in non-overlapping blocks of size 32×32.
2) Prediction parameters are estimated using Jacobian Iteration method for each non-overlapping block and prediction of unfilled pixels at even-even positions are done using estimated prediction parameters.
3) Estimation of Jacobian prediction parameters (α_1, α_2, α_3, and α_4) is explained in two steps which is as follows.

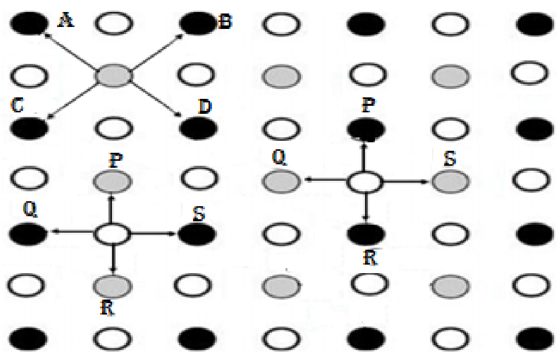

Fig. 2. Neighboring pixels (A,B,C,D and P,Q,R,S) used for interpolation in First and Second Phase respectively.

1) Estimation of Covariance matrix for each block of LR image: In this step, we divide the source image S into non-overlapping blocks of size 16 X 16. Learning the local characteristics of each block gives the optimized prediction coefficients to be used in prediction of missing pixels of HR image.

Suppose $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the optimal unknown prediction coefficients to predict pixel $X_{LR}(n)$ using neighboring pixels A,B,C and D respectively. Then the predicted pixel $\hat{X}_{LR}(n)$

can be written as:

$$\tilde{X}_{LR}(n) = \alpha_1 A + \alpha_2 B + \alpha_3 C + \alpha_4 D \quad (4)$$

where $\tilde{X}_{LR}(n)$ is predicted LR pixels. Thus the error square found in the prediction is

$$e^2(i,j) = \{X_{LR}(n) - \tilde{X}_{LR}(n)\}^2. \quad (5)$$

The Square Error of each pixel of a block in LR images are accumulated and minimization of error is done which give rise to covariance matrix [12].

$$\begin{bmatrix} \Sigma A^2 & \Sigma AB & \Sigma AC & \Sigma AD \\ \Sigma AB & \Sigma B^2 & \Sigma BC & \Sigma BD \\ \Sigma AC & \Sigma BC & \Sigma C^2 & \Sigma CD \\ \Sigma AD & \Sigma BD & \Sigma CD & \Sigma D^2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} \Sigma XA \\ \Sigma XB \\ \Sigma XC \\ \Sigma XD \end{bmatrix}$$

We can see that the above covariance matrix is a kind of linear equation of the form $Ax = B$.

Conventional methods for solving these equations involve matrix inversion resulting in higher computational cost which withdraws it from the race of real time application. Moreover, they don't work in situations where the matrix A comes out to be singular which is generally observed in the medical images or some dark shade natural images like Fish1,Fish2, Fish3 as shown in Fig. 4. Thus, Jacobi Iteration method is used to solve the above mentioned problem and is follows.

2) Application of Jacobi iteration method to get optimal prediction coefficients:

1) We use the Jacobi iteration method to get the predictors (α_1, α_2, α_3, and α_4) with the help of equation (1).
2) Pseudo code for the calculation of predictors is also given in the Fig. 3.
3) Keeping the value of predictors $\alpha_i^{(k)}$ (for k = 0 only , $1 \leq i \leq 4$) as 0.25, initially acting as feedback helps us to get the desired result in less number of iteration as it is witnessed that Jacobi starts diverging in case of matrices other than diagonally dominant.
4) Using coefficients of bilinear ($\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$, for k=0) as feedback results in reducing the computational cost to larger extent by decreasing the iteration steps.
5) Involvement of lesser number of iteration steps enables the use of Jacobian Iteration method in spite of its diverging properties for general set of matrices.
6) Thus from the prediction coefficients for each block generated by Jacobian iteration method all the missing pixels of HR image categorized in phase I are calculated using the equation.

$$\tilde{X}_{HR}(n) = \alpha_1 A + \alpha_2 B + \alpha_3 C + \alpha_4 D. \quad (6)$$

```
for J = 1 to N     %% N is the maximum numer of iteration.

  xnew(1) = ( b(1) - sum ( A(1,2:N) .* xold(2:N) ) ) / A(1,1);

     for I = 2 to N-1

     xnew(I) = ( b(I) - sum ( A(I,1:I-1) .* xold(1:I-1) )

               - sum ( A(I,I+1:N) .* xold(I+1:N) ) ) / A(I,I);

     end;

     xnew(N) = ( b(N) - sum ( A(N,1:N-1) .* xold(1:N-1) ) ) / A(N,N);


     var = max ( abs ( xnew - xold ) );

     if ( var < TOL )  %% TOL is the tolerance value which determines
                       %% the convergence of the residual values.
        x = xnew;
        return

     else
        xold = xnew;
     end;
end;
```

Fig. 3. Pseudo Code explaining the the estimation of predictor coefficients.

B. Phase-II

The prediction of missing pixels in the second phase with at least one odd spatial coordinate requires the same steps as in phase I but the neighboring pixels involved in prediction get changed according to availability.

1) Estimation of covariance matrix using Least Square Estimation.
2) Solving covariance matrix using Jacobi Iteration method.
3) Thus rest of the missing pixels in phase II are predicted with the help of Jacobian parameters (β_1, β_2, β_3, and β_4) using the equation

$$\tilde{Y}_{HR}(n) = \beta_1 P + \beta_2 Q + \beta_3 R + \beta_4 S. \quad (7)$$

In this way all the missing pixels of High resolution images are predicted using the Least Square error based Jacobian parameters (α_1, α_2, α_3,α_4 , β_1, β_2, β_3, and β_4).

IV. Simulation Results

In order to validate the performance of our proposed algorithm, several comparisons have been made with the existing interpolation methods like bilinear interpolation, Content adaptive interpolation [2] and Context based image independent interpolation [5] scheme. Although [4],[6] and [7] gives better results than our algorithm, however we are giving detailed analysis of competitive methods (Computationally as simple as ours) in Table I. Results for bilinear interpolation have been obtained using mat lab code in [11].

PSNR of all the test images[10] in Fig. 4 are calculated from the above mentioned methods which are tabulated in Table I. Our proposed algorithm on an average gives 0.24, 0.54 and 0.16 db better PSNR than Bilinear, CAI and CBII respectively.

TABLE I
PSNR COMPARISON OF INTERPOLATED IMAGES BY DIFFERENT METHODS.

Images	Bilinear	CAI [2]	CBII	Proposed
Cycle	20.356	20.03	20.4181	**20.537**
Fish1	27.258	26.858	27.528	**27.690**
Fish2	25.528	25.387	25.571	**25.786**
Fish3	27.367	27.238	27.369	**27.496**
Girl	33.303	33.380	33.437	**33.535**
Barbara	25.015	24.366	24.81	**25.101**
Fruits	21.002	20.691	21.05	**21.196**
Island	24.962	24.362	25.07	**25.20**
Home	24.351	24.118	24.499	**24.604**
Forest	26.709	26.41	26.914	**27.054**
Avg	25.58	25.284	25.66	**25.82**

TABLE II
CORRELATION COEFFICIENT OF THE HR IMAGES USING THE MENTIONED INTERPOLATION ALGORITHMS

Images	Bilinear	CAI [2]	CBII	Proposed
Cycle	0.9444	0.9391	0.9445	**0.9451**
Fish1	0.9707	0.9655	0.9711	**0.9716**
Fish2	0.9800	0.9784	0.9797	**0.9803**
Fish3	0.9868	0.9864	0.9870	**0.9871**
Girl	0.9941	0.9931	**0.9943**	**0.9943**
Barbara	**0.9658**	0.9595	0.9638	0.9655
Fruits	0.9340	0.9295	0.9355	**0.9365**
Island	0.9315	0.9194	0.9315	**0.9317**
Home	0.9695	0.9675	0.9709	**0.9710**
Forest	0.9704	0.9670	0.9707	**0.9711**
Avg	0.9647	0.9605	0.9649	**0.9654**

Another approach for the validation of the performance of our proposed algorithm is the comparison of the Correlation Coefficients **C** (8) of the HR images tabulated in table II. The Correlation Coefficient C gives value between 0 and 1. If C is more closer to 1, then the interpolated image is more similar to the original high-resolution image. Although the CBII and proposed method in Table II gives similar Correlation Coefficient in case of test image "Girl" but it could be seen in Table I that the proposed algorithm obtain the better PSNR result than CBII.

$$C = |\frac{\Sigma O(i,j)I(i,j) - WHUV}{\sqrt{(\Sigma O^2(i,j) - WHUV)(\Sigma I^2(i,j) - WHUV)}}| \quad (8)$$

where U and V are the mean pixel values of the original image O and the interpolated image I. In (8), H and W are the height and width of the interpolated image I, whereas O(i,j) and I(i,j) are the pixels values of original image and interpolated image I respectively.

Fig. 4. Test Images of size 512×512

V. CONCLUSION

In this paper, an algorithm with lower computational cost has been proposed where the prediction scheme using the local characteristics of images. The use of Jacobian method to solve the covariance matrix from LR image, reduces the computational power and gives the optimal predictors on the basis of the behavior of the neighboring pixels. The experimental data shows that the proposed algorithm with marginal increment of the computational cost provides significant improvement in objective results when compared with classical interpolation schemes.

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